UNIT

6

DESIGN OF CLUTCHES AND BRAKES

6.1 CLUTCH

A clutch is a device used to connect or disconnect the driving and driven shafts at will without interfering the driving shaft. The flow of mechanical power is controlled by means of a clutch.

Clutches are mainly classified into two types; (i) Positive clutches & (ii) Friction clutches

In positive clutches the engaging clutch surfaces interlock to produce a rigid joint. They are usually of jaw type. The jaws may be either square cross section, saw tooth type, spiral profile type or gear-tooth type. The cross-section of the jaws may be determined by considering their failure in shear and crushing. The merits of positive clutches are (i) No slip (ii) No heat generated (iii) Compact and low cost.

The friction type of clutches work on the basis of the frictional forces developed between the surfaces in contact. Friction clutches are usually preferred over the jaw clutches due to their better performance. There is slip in friction clutch. The merits of friction clutches are (i) They can slip during engagement which enables the driver to pickup and accelerate the load with minimum shock. (ii) They can be used at high engagement speeds since they do not have jaws or teeth. (iii) Smooth engagement due to the gradual increase in normal force. Friction clutches are mainly classified as (i) Single plate clutch (ii) Multiplate clutch (iii) Cone clutch (iv) Centrifugal clutch.

6.2 SINGLE PLATE CLUTCH

A single plate clutch is shown in Fig. 6.1 consists of two flanges, one is rigidly keyed to the driving shaft by means of a sunk key and the other is fitted to the driven shaft by a feather key or spline so that it can be moved along the shaft. The face of the driven flange is lined with friction material such as leather ferrodo cork or asbestos. The actuating force is provided by a spring which forces the driven flange to move towards the driving flange. Torque is transmitted by means of frictional force between the driving and driven flanges. In single plate friction clutch, both sides of the clutch plates are faced with frictional material such as ferrodo.

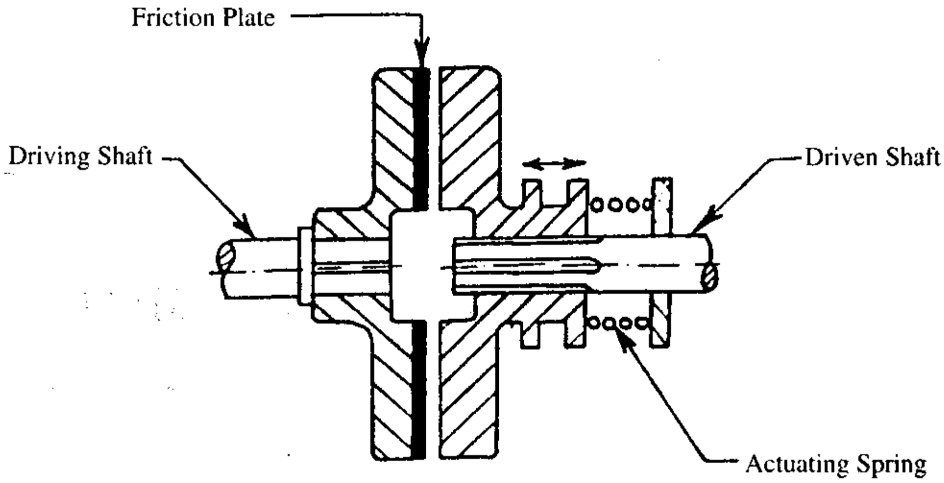


Fig. 6.1 : Single plate clutch

6.3 MULTI-PLATE CLUTCH

A multi-plate clutch shown in Fig. 6.2 is normally used for driving boats by internal combustion engines. The driving disks and the driven disks are comparatively thin cast iron plates without any facing material and they run in an oil bath. The driving disks have square holes fitted to the square shaped hub. The driven plates are connected to the housing by pins.

The number of pairs of contacting surfaces in a single plate clutch is one or two and in multi-plate it is more than two. For a given torque capacity, the size of a multi-plate clutch is smaller than that of a single plate clutch. Multi-plate clutches are wet clutches where as single plate clutches are dry clutches. Single plate clutches are used where large radial space is available such as trucks and cars. Multi-disk clutches are used where compact construction is desirable e.g. scooters.

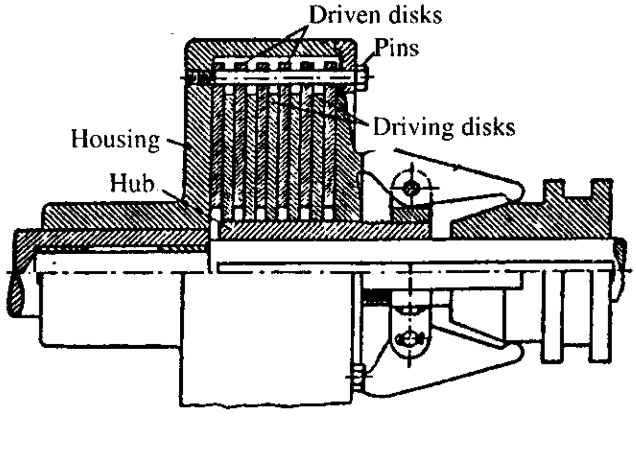


Fig. 6.2 : Multi disk marine-type clutch

6.4 TORQUE TRANSMITTED BY DISC OR PLATE CLUTCH

Figure 6.3 shows a friction disk of a single plate clutch.

Let M_t = Torque transmitted by friction in Nmm

F_a = Axial force in N

μ = Coefficient of friction

p = Intensity of normal pressure between the surfaces N/mm^2

R_1 = Inner radius of friction surface

R_2 = Outer radius of friction surface

D_1 = Inner diameter of friction surface

D_2 = Outer diameter of friction surface

R_m = Mean radius of friction surface

D_m = Mean diameter of friction surface

N = Power transmitted in kW

n = Speed in rpm

d = Diameter of shaft

η = Keyway factor

i = Number of active surfaces = $i_1 + i_2 - 1$

i_1 = Number of driving discs = i/2

i_2 = Number of driven discs = i/2 + 1

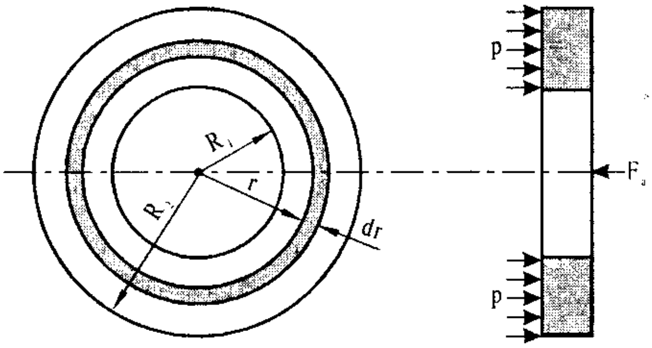


Fig. 6.3

Consider an elemental ring of radius r and radial thickness dr as shown in Fig. 6.3.

Area of elemental ring = $2\pi r\, dr$

Axial force on the elemental ring = $(2\pi r\, dr)\, p$

$$\therefore \text{ Total axial force } F_a = \int_{R_1}^{R_2} 2\pi p r\, dr \qquad \text{---- (i)}$$

Friction force on the elemental ring = μ p 2 π r dr

Moment of friction force about the axis or frictional torque on the elemental ring = $(\mu p 2\pi r dr) r$

Total frictional torque per active surface $M_t = \int_{R_1}^{R_2} 2\mu\pi p r^2 dr$ (ii)

There are two cases to obtain the torque capacity (i) uniform pressure (ii) uniform wear

(i) Uniform pressure theory

According to this criterion, it is assumed that the pressure is uniformly distributed over the entire surface area of the friction disk. This assumption is used for new clutches employing number of springs. Since pressure is uniformly distributed, p is constant

From equation (i)

Total axial force $F_a = \int_{R_1}^{R_2} 2\pi p r dr = 2\pi p \int_{R_1}^{R_2} r dr = 2\pi p \left(\frac{r^2}{2}\right)_{R_1}^{R_2} = \pi p (R_2^2 - R_1^2)$

$\therefore$ Axial force $F_a = \pi p \left(\frac{D_2^2 - D_1^2}{4}\right)$ ---- 19.86.b (DDHB)

From equation (ii)

Total frictional torque per active surface $M_t = \int_{R_1}^{R_2} 2\pi\mu p r^2 dr$

$$= 2\pi\mu p \int_{R_1}^{R_2} r^2 dr = 2\pi\mu p \left(\frac{R_2^3 - R_1^3}{3}\right)$$

$$= 2\pi\mu . \frac{F_a}{\pi(R_2^2 - R_1^2)} . \left(\frac{R_2^3 - R_1^3}{3}\right) \quad \{\because F_a = \pi p (R_2^2 - R_1^2)\}$$

$$= \frac{2}{3}\mu F_a \left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right) = \frac{\mu F_a}{2} . \frac{2}{3}\left[\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right]$$

i.e., $M_t = \frac{1}{2}\mu F_a D_m$ where D_m = Mean diameter ---- 19.84 (DDHB)

$D_m = \frac{2}{3}\left[\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right]$ ---- 19.85 (a) (DDHB)

Torque transmitted by i friction surfaces $M_t = \frac{1}{2}\mu F_a D_m . i$

(ii) Uniform wear theory

According to this criterion, it is assumed that the wear is uniformly distributed over the entire surface area of the disk. This assumption is used for worn-out clutches. Rate of wear is proportional to pv where v is rubbing velocity since $v = \omega r$

Rate of wear α pr. i.e., pr = constant = c

From equation (i)

$$\text{Total axial force } F_a = \int_{R_1}^{R_2} 2\pi prdr = 2\pi c \int_{R_1}^{R_2} dr$$

$$\therefore \quad F_a = 2\pi c(R_2 - R_1)$$

From equation (ii)

$$M_t = \int_{R_1}^{R_2} 2\pi\mu pr^2 dr = 2\pi\mu c \int_{R_1}^{R_2} rdr = 2\pi\mu c\left(\frac{r^2}{2}\right)_{R_1}^{R_2}$$

$$= \pi\mu c(R_2^2 - R_1^2)$$

$$= \pi\mu\left(\frac{F_a}{2\pi(R_2 - R_1)}\right)(R_2^2 - R_1^2) \quad [\because F_a = 2\pi c(R_2 - R_1)]$$

$$= \frac{1}{2}\mu F_a(R_2 + R_1) = \mu F_a R_m$$

$$\therefore \quad M_t = \frac{1}{2}\mu F_a D_m \text{ where } D_m = \text{Mean diameter} = \frac{D_2 + D_1}{2} \quad \text{---- 19.84 (DDHB)}$$

Torque transmitted for i friction surfaces $M_t = \frac{1}{2}\mu F_a D_m \cdot i$

$$\text{Axial force } F_a = 2\pi c(R_2 - R_1) = 2\pi pr(R_2 - R_1)$$

Since maximum pressure occurs at the inner radius, $F_a = 2\pi pR_1(R_2 - R_1)$

$$\text{i.e., } F_a = \frac{1}{2}\pi pD_1(D_2 - D_1) \quad \text{---- 19.83 (DDHB)}$$

For average pressure

$$F_a = 2\pi pR_m(R_2 - R_1) = \frac{1}{2}\pi pD_m(D_2 - D_1)$$

Note :

Since the uniform pressure theory gives a higher friction torque than the uniform wear theory, the uniform wear theory should always be used unless otherwise stated.

6.5 FRICTION MATERIALS

Leather, wood or cork are used as friction materials for light load and low speeds. Two types of friction disks are commonly used (i) asbestos-base (ii) sintered metals. There are two types of asbestos friction materials (i) woven (ii) moulded. The difference between woven and moulded asbestos materials are,

(i) Woven material is flexible, while moulded asbestos is rigid.

(ii) Woven material has higher coefficient of friction.

(iii) Woven material are costly and wear at a faster rate.

There are two types of sintered friction materials (i) bronze base (ii) iron base. The merits of sintered friction disks are (i) It can be used for high temperatures. (ii) Higher wear resistance (iii) Coefficient of friction is constant.

A good friction material should have the following properties

i. A high coefficient of friction capable of remaining uniform over a wide range of surface velocities, temperatures and loads.
ii. Adequate mechanical and thermal strength
iii. Little wear and no scoring.
iv. High heat conductivity for rapid transfer of heat from the friction surfaces.

6.6 DESIGN PROCEDURE

The design consideration mainly depends upon, how to obtain the best balance between maximum torsional moment capacity and effective operating life. The following informations must be available. (i) Torsional moment to be transmitted (ii) Limiting dimensions (iii) Pressure required to engage the clutch plates. (iv) Single disc or multi disc construction.

Steps

1. Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} \qquad \text{----- 19.3 c (DDHB)}$$

2. Diameter of shaft

$$d = \sqrt[3]{\frac{16M_t}{\pi\tau_s\eta}} \qquad \text{----- 19.49 (DDHB)}$$

3. Mean diameter

For uniform pressure condition $D_m = \frac{2}{3}\left[\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right]$ ---- 19.85 a (DDHB)

For uniform wear condition $D_m = \frac{D_2 + D_1}{2}$ ---- 19.85 b (DDHB)

4. Axial force

For uniform pressure condition :

$$F_a = \pi p\left(\frac{D_2^2 - D_1^2}{4}\right) \qquad \text{---- 19.86 b (DDHB)}$$

For uniform wear condition

$$F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) \qquad \text{---- 19.83 (DDHB)}$$

5. Dimensions of clutch

Also torque transmitted $M_t = \frac{1}{2}\mu F_a D_m i$ ---- 19.84 (DDHB)

Inner diameter $= D_1$

Outer diameter $= D_2$

Mean diameter $= D_m$

Thickness of disc $h = 1$ to 3 mm ---- 19.92 (DDHB)

Axial force $= F_a$

Example 6.1

A car engine develops maximum power of 15 kW at 1000 rpm. The clutch used is single plate type of both sides effective having external diameter 1.25 times internal diameter μ = 0.3. Mean axial pressure is not to exceed 0.085 N/mm². Determine the dimensions of the friction surface and the force necessary to engage the plates. Assume uniform pressure condition.

Data :

$N = 15$ kW; $n = 1000$ rpm; $i = 2$ (since both sides are effective); $D_2 = 1.25\,D_1$; $\mu = 0.3$;
$p = 0.085$ N/mm² uniform pressure condition.

Solution :

i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{15}{1000} = 143250 \text{ Nmm}$$

ii) Mean diameter

For uniform pressure condition

$$D_m = \frac{2}{3}\left(\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right) = \frac{2}{3}\left[\frac{(1.25D_1)^3 - D_1^3}{(1.25D_1)^2 - D_1^2}\right] = 1.13\,D_1$$

iii) Axial force

For uniform pressure condition

$$\text{Axial force } F_a = \pi p\left(\frac{D_2^2 - D_1^2}{4}\right) = \pi \times 0.085\,\frac{\{(1.25D_1)^2 - D_1^2\}}{4} = 0.037552\,D_1^2$$

iv) Dimensions

Also torque transmitted $M_t = \frac{1}{2}\,\mu F_a D_m\, i$

$$\text{i.e., } 143250 = \frac{1}{2} \times 0.3 \times 0.037552\, D_1^2 \times 1.13\, D_1 \times 2$$

$\therefore$ Inner diameter of friction surface $D_1 = 224$ mm

Outer diameter of friction surface $D_2 = 280$ mm ($\because D_2 = 1.25\, D_1$)

Mean diameter of friction surface $D_m = 253$ mm ($\because D_m = 1.13\, D_1$)

Axial force $F_a = 1884.21$ N ($\because F_a = 0.037552\, D_1^2$)

Thickness of disc $h = 2$ mm ----- 19.92 (DDHB)

Example 6.2

Design a single plate clutch consists of two pairs of contacting surfaces for a torque capacity of 200 Nm. Due to space limitations the outside diameter of the clutch is to be 250 mm. VTU, June/July'08

Data :

Single plate clutch; $M_t = 200$ Nm $= 2 \times 10^5$ Nmm; $D_2 = 250$ mm; $i = 2$ (since two pairs of contacting surfaces)

Solution :

Assume leather as the friction material $\therefore$ From Table 19.7 for leather $\mu = 0.3$ to 0.5 and $p = 0.0686$ to 0.2746 MPa

Select $\mu = 0.4$ and $p = 0.135$ N/mm^2

i) Torque transmitted

$$M_t = 2 \times 10^5 \text{ Nmm (given)}$$

ii) Mean diameter

Assume uniform wear condition

$$\therefore D_m = \frac{1}{2}(D_2 + D_1) = \frac{1}{2}(250 + D_1)$$

iii) Axial force

For uniform wear condition

$$\text{Axial force } F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) = \frac{1}{2} \times \pi \times 0.135 \times D_1 (250 - D_1)$$

$$= 0.212\, D_1 (250 - D_1)$$

iv) Dimensions

Also torque transmitted $M_t = \frac{1}{2}\,\mu F_a D_m\, i$

$$\text{i.e., } 2 \times 10^5 = \frac{1}{2} \times 0.4 \times 0.212\, D_1 (250 - D_1) \times \frac{1}{2}(250 + D_1) \times 2$$

i.e., $62500\, D_1 - D_1^3 - 4716981.132 = 0$

By trial and error method

$$D_1 = 85.46 \text{ mm} \approx 86 \text{ mm}$$

i.e., Inner diameter of friction surface $D_1 = 86$ mm

Outer diameter of friction surface $D_2 = 250$ mm (given)

Mean diameter of friction surface $D_m = 168$ mm

$$\text{Axial force } F_a = 0.212 \times 86\,[250 - 86] = 2990 \text{ N}$$

$$\text{Thickness of disc } h = 2 \text{ mm}$$

Example 6.3

Determine the power transmitted by a single pair plate clutch assuming uniform pressure distribution. The friction surfaces have an outside diameter of 350 mm and an inner diameter of 280 mm. The coefficient of friction is 0.25 and the maximum allowable pressure is 0.85 MPa.

VTU July/August 2002

Data :

$i = 1$ ($\because$ Single pair contact); uniform pressure distribution; $D_2 = 350$ mm; $D_1 = 280$ mm; $\mu = 0.25$; $p = 0.85$ N/mm^2

Solution :

For uniform pressure condition

$$\text{Axial force } F_a = \pi p\left(\frac{D_2^2 - D_1^2}{4}\right) = \pi \times 0.85\left(\frac{350^2 - 280^2}{4}\right) = 29440.65 \text{ N}$$

For uniform pressure condition

$$\text{Mean diameter } D_m = \frac{2}{3}\left(\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right) = \frac{2}{3}\left(\frac{350^3 - 280^3}{350^2 - 280^2}\right) = 316.3 \text{ mm}$$

Torque transmitted by one pair of friction surfaces (i.e., one active surface) $M_t = \frac{1}{2}\mu F_a D_m$

$$= \frac{1}{2} \times 0.25 \times 29440.65 \times 316.3 = 116399.6 \text{ Nmm}$$

$$\text{Also } M_t = 9550 \times 1000 \times \frac{N}{n}$$

Since speed is not given assume n = 1000 rpm

$$\therefore\ 116399.6 = 9550 \times 1000 \times \frac{N}{1000}$$

$\therefore$ Power transmitted $N = 12.2$ kW

Example 6.4

Design a single plate clutch used in automobile transmission for the following specifications Power to be transmitted = 20 kW; speed 1500 rpm to 2500 rpm (max) Friction surface moulded asbestos on steel.

Data :

n_1 = 1500 rpm = Minimum speed; n_2 = 2500 rpm = Maximum speed; N = 20 kW;
Friction surface – moulded asbestos on steel.

Solution :

Assume both sides of the friction plate is effective $\therefore$ i = 2

From Table 19.7 for moulded asbestos on steel μ = 0.2 to 0.5 and p = 0.3452 to 1.0346 MPa

Select μ = 0.35 and p = 1 N/mm². Assume C40 steel as shaft material and factor of safety = 3

$\therefore$ From Table 1.5 (Old DDHB) for C40 steel

$$\sigma_y = 328.6 \text{ MPa}$$

$$\therefore \sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{3} = 109.53 \text{ N/mm}^2$$

$$\tau = 0.5\,\sigma = 0.5 \times 109.53 = 54.77 \text{ N/mm}^2 = \tau_s$$

Assume key way factor $\eta = 0.75$

i) Torque transmitted

Design for maximum torque

$$\therefore M_t = 9550 \times 1000 \times \frac{N}{n_1} = 9550 \times 1000 \times \frac{20}{1500} = 127333.33 \text{ Nmm}$$

ii) Diameter of shaft

$$d = \sqrt[3]{\frac{16M_t}{\pi \tau_s \eta}} = \sqrt[3]{\frac{16 \times 127333.33}{\pi \times 54.77 \times 0.75}} = 25.08 \text{ mm}$$

Standard diameter of shaft d = 30 mm [From Table 14.6]

iii) Mean diameter

Assume uniform wear condition

$$D_m = \frac{1}{2}(D_2 + D_1)$$

Inner diameter of friction surface $D_1 = 4\,d = 4 \times 30 = 120$ mm ---- 19.91 (DDHB)

$$\therefore D_m = \frac{1}{2}(D_2 + 120)$$

iv) Axial force

For uniform wear condition

$$F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) \quad \text{---- 19.83 (DDHB)}$$

$$= \frac{1}{2}\pi \times 1 \times 120\,[D_2 - 120] = 188.5\,(D_2 - 120)$$

v) Dimensions

$$\text{Also torque transmitted } M_t = \frac{1}{2}\mu F_a D_m \times i$$

$$\text{i.e., } 127333.33 = \frac{1}{2} \times 0.35 \times 188.5 (D_2 - 120) \times \frac{1}{2}(D_2 + 120) 2$$

$$3860.05 = D_2^2 - 14400$$

$$D_2 = 135.13 \text{ mm}$$

Select outer diameter of friction surface $D_2 = 140$ mm

$$\text{Axial force } F_a = 188.5 [D_2 - 120] = 188.5 [140 - 120]$$

$$= 3770 \text{ N}$$

Thickness of friction disc $h = 2$ mm ---- 19.92 (DDHB)

Example 6.5

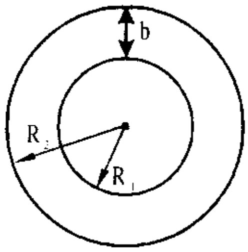
In a multiple disc clutch the radial width of the friction material is to be 0.2 of maximum radius. The coefficient of friction is 0.25. The clutch is to transmit 60 kW at 3000 rpm. Its maximum diameter is 250 mm and the axial force is limited to 600 N. Determine (i) Number of driving and driven discs. (ii) Mean unit pressure on each contact surface. Assume uniform wear.

VTU July/August 2003; BU February 1995

Data :

Radial width $b = 0.2 R_2$; $\mu = 0.25$ N = 60 kW; n = 3000 rpm; $D_2 = 250$ mm ∴ $R_2 = 125$ mm
$F_a = 600$ N; Uniform wear condition.

Solution :



$$b = R_2 - R_1$$

$$\text{i.e., } 0.2 R_2 = R_2 - R_1$$

$$\therefore R_1 = 0.8 R_2 = 0.8 \times 125 = 100 \text{ mm}$$

∴ Inner diameter of friction surface $D_1 = 200$ mm

i) Number of discs

$$\text{Torque transmitted } M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{60}{3000} = 191000 \text{ Nmm}$$

For uniform wear condition,

$$\text{Mean diameter } D_m = \frac{1}{2}(D_2 + D_1) = \frac{1}{2}(250 + 200) = 225 \text{ mm}$$

$$\text{Also torque transmitted } M_t = \frac{1}{2}\mu F_a D_m i$$

$$\text{i.e., } 191000 = \frac{1}{2} \times 0.25 \times 600 \times 225 \times i$$

$$\text{i.e., } i = 11.32$$

∴ Number of active surfaces $i = 12$ (i must be an even number)

Number of discs on the driver shaft $i_1 = \frac{i}{2} = \frac{12}{2} = 6$ ---- 19.94 (DDHB)

Number of discs on the driven shaft $i_2 = \frac{i}{2} + 1 = \frac{12}{2} + 1 = 7$ ---- 19.95 (DDHB)

$\therefore$ Total number of discs $= i_1 + i_2 = 6 + 7 = 13$

ii) Mean unit pressure

For limiting mean unit pressure

$$F_a = \frac{1}{2} \pi p D_m (D_2 - D_1)$$

$$\text{i.e., } 600 = \frac{1}{2} \times \pi \times p \times 225 \times (250 - 200)$$

$$\therefore \mathbf{p = 0.034\ N/mm^2}$$

For actual mean unit pressure

$$\text{Actual axial force } F_a = \frac{2M_t}{\mu i D_m} \quad (\because M_t = \frac{1}{2} \mu F_a D_m i)$$

$$= \frac{2 \times 191000}{0.25 \times 12 \times 225} = 565.926 \text{ N/mm}^2$$

$$\therefore 565.926 = \frac{1}{2} \times \pi \times p \times 225 \times (250 - 200)$$

$$\therefore \text{Actual mean unit pressure } p = 0.032 \text{ N/mm}^2$$

Example 6.6

A 25 kW at 3000 rpm is to be transmitted by a multiplate friction clutch. The plates have friction surfaces of steel and phosphor bronze alternatively and run in oil. Design the clutch for 25% over load.

Data :

N = 25 kW; n = 3000 rpm; Overload = 25% Friction material (Steel and phosphor bronze).

Solution :

From Table 19.7 for steel and phosphor bronze, $\mu = 0.03$ and $p = 1.0346$ N/mm^2

Assume C30 steel for shaft and factor of safety = 3

From Table 1.5 (Old DDHB) for C30 steel

$$\sigma_y = 294.2 \text{ MPa} \quad \therefore \sigma = \frac{\sigma_y}{FOS} = \frac{294.2}{3} = 98.06 \text{ N/mm}^2$$

$$\tau = 0.5\,\sigma = 0.5 \times 98.06 = 49.03 \text{ N/mm}^2$$

$$\text{Assume key way factor } \eta = 0.75$$

i) Torque transmitted

Design for maximum torque

$$M_t = 9550 \times 1000 \times \frac{N}{n} \times \text{over load}$$

$$= 9550 \times 1000 \times \frac{25}{3000} \times 1.25 = 99479.167 \text{ Nmm}$$

ii) Diameter of shaft

$$d = \sqrt[3]{\frac{16M_t}{\pi\tau_s.\eta}} = \sqrt[3]{\frac{16 \times 99479.167}{\pi \times 49.03 \times 0.75}} = 23.97 \text{ mm}$$

Standard diameter of shaft $d = 25$ mm [From Table 14.6]

iii) Mean diameter

Assume uniform wear condition $D_m = \frac{1}{2}(D_2 + D_1)$

Inner diameter of friction surface $D_1 = 4d = 4 \times 25 = 100$ mm ---- 19.91 (DDHB)

Outer diameter of friction surface $D_2 = 1.25\ D_1$ to $1.8\ D_1$ ---- 19.90 (DDHB)

Take $D_2 = 1.5\ D_1 = 1.5 \times 100 = 150$ mm

$\therefore\ D_m = \frac{1}{2}(150 + 100) = 125$ mm

iv) Axial force

For uniform wear condition

$$F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) = \frac{1}{2}\pi \times 1.0346 \times 100 \times [150 - 100] = 8125.73 \text{ N}$$

v) Number of discs

Also torque transmitted $M_t = \frac{1}{2}\mu F_a D_m i$

i.e., $99479.167 = \frac{1}{2} \times 0.03 \times 8125.73 \times 125 \times i$

$i = 6.529$

$\therefore$ Number of active surfaces $i = 8$ ($\because$ i must be an even number)

Number of discs on the driver shaft $i_1 = \frac{i}{2} = \frac{8}{2} = 4$

Number of discs on the driven shaft $i_2 = \frac{i}{2} + 1 = \frac{8}{2} + 1 = 5$

$\therefore$ Total number of discs $= i_1 + i_2 = 4 + 5 = 9$

Thickness of each friction disc $h = 1.5$ mm ---- 19.92 (DDHB)

Example 6.7

A multiple plate clutch has steel on bronze is to transmit 8 kW at 1440 rev/min. The inner diameter of the contact is 80 mm and the outer diameter of contact is 140 mm. The clutch operates in oil with expected coefficient of friction of 0.1, the average allowable pressure is 0.35 MPa. Assume uniform wear theory and determine the following.

a) Number of steel and bronze plates b) Axial force required c) Actual maximum pressure

VTU February 2002; July/August 2004. Dec'06/Jan'07

Data:

$N = 8$ kW; $n = 1440$ rpm; $D_1 = 80$ mm; $D_2 = 140$ mm; $\mu = 0.1$; $p = 0.35$ N/mm²; Uniform wear theory.

Solution :

a) Number of steel and bronze plates

For uniform wear theory

$$\text{Axial force } F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) = \frac{1}{2} \times \pi \times 0.35 \times 80\,[140 - 80] = 2638.94 \text{ N}$$

For uniform wear theory, mean diameter $D_m = \frac{1}{2}(D_2 + D_1) = \frac{1}{2}(140 + 80) = 110$ mm

$$\text{Torque transmitted } M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{8}{1440} = 53055.556 \text{ Nmm}$$

$$\text{Also } M_t = \frac{1}{2}\mu F_a D_m i$$

$$\text{i.e., } 53055.556 = \frac{1}{2} \times 0.1 \times 2638.94 \times 110 \times i$$

$$i = 3.655$$

$\therefore$ Number of active surfaces $i = 4$

Number of discs on the driver shaft (bronze plate) $i_1 = \frac{i}{2} = \frac{4}{2} = 2$

Number of discs on the driven shaft (steel plate) $i_2 = \frac{i}{2} + 1 = \frac{4}{2} + 1 = 3$

$\therefore$ Total number of discs $i_1 + i_2 = 2 + 3 = 5$

b) Axial force required

Actual axial force $F_a = \frac{2M_t}{\mu i D_m} = \frac{2 \times 53055.556}{0.1 \times 4 \times 110} = 2411.62$ N

c) Actual maximum pressure

Since maximum pressure occurs at the inner radius

$$F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1)$$

$$\text{i.e., } 2411.62 = \frac{1}{2} \times \pi \times p \times 80 \times (140 - 80)$$

$$\therefore \; p = 0.32 \text{ N/mm}^2$$

Example 6.8

A multiple clutch has 2 bronze and 3 steel discs. The friction material can withstand a pressure of 0.1 N/mm² and $\mu = 0.15$. The outside and inside diameters of friction lining are 200 mm and 120 mm respectively. Determine the power that can be transmitted by this clutch at 1000 rpm.

Data :

$i_1 = 2$; $i_2 = 3$; $p = 0.1$ N/mm²; $\mu = 0.15$; $D_2 = 200$ mm; $D_1 = 120$ mm; $n = 1000$ rpm

Solution :

Assume uniform wear theory

$$\therefore \quad \text{Mean diameter } D_m = \frac{1}{2}(D_2 + D_1) = \frac{1}{2}(200 + 120) = 160 \text{ mm}$$

$$\text{Axial force } F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) = \frac{1}{2} \times \pi \times 0.1 \times 120\,(200 - 120) = 1507.96 \text{ N}$$

$$\text{Number of active surfaces } i = i_1 + i_2 - 1 = 2 + 3 - 1 = 4 \qquad \text{---- 19.93 (DDHB)}$$

$$\text{Torque transmitted } M_t = \frac{1}{2}\mu F_a D_m i = \frac{1}{2} \times 0.15 \times 1507.96 \times 160 \times 4 = 72382.03 \text{ Nmm}$$

$$\text{Also } M_t = 9550 \times 1000 \times \frac{N}{n}$$

$$\text{i.e., } 72382.03 = 9550 \times 1000 \times \frac{N}{1000}$$

$$\therefore \quad \text{Power transmitted } N = 7.58 \text{ kW}$$

6.7 CONE CLUTCH

A simple form of cone clutch is shown in Fig. 6.4. It consists of a driver or cup and a follower or cone. The outer cone or cup is keyed to the driving shaft by means of a sunk key, while the inner cone or follower is free to slide axially on the driven shaft due to splines. The axial force required to engage the clutch is provided by a helical compression spring. A fork is used in shifting collar to disengage the clutch. Leather, asbestos or cork are used as friction lining on the inner cone. The semi-cone angle α is always kept greater than the angle of static friction to avoid self engagement. The preferable semi-cone angle is 12.5.

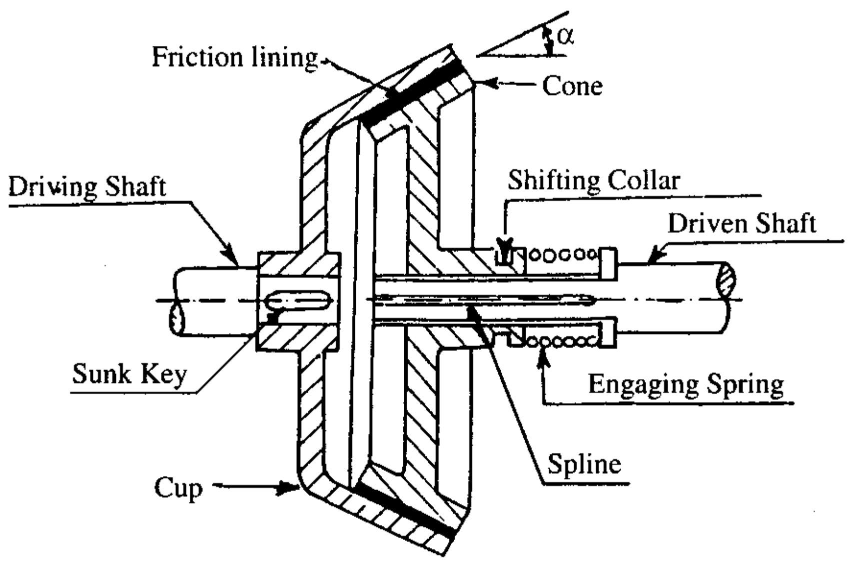


Fig. 6.4 : Cone clutch

6.8 MERITS AND DEMERITS OF CONE CLUTCH

Merits

i) Cone clutches are easy to disengage and simple in construction.

ii) Less axial force is required to engage the clutch.

Demerits

i) Strict requirement for the coaxiality of two shafts.

ii) There is a tendency to grap.

6.9 TORQUE TRANSMITTED BY CONE CLUTCH

Consider a friction cone as shown in Fig. 6.5

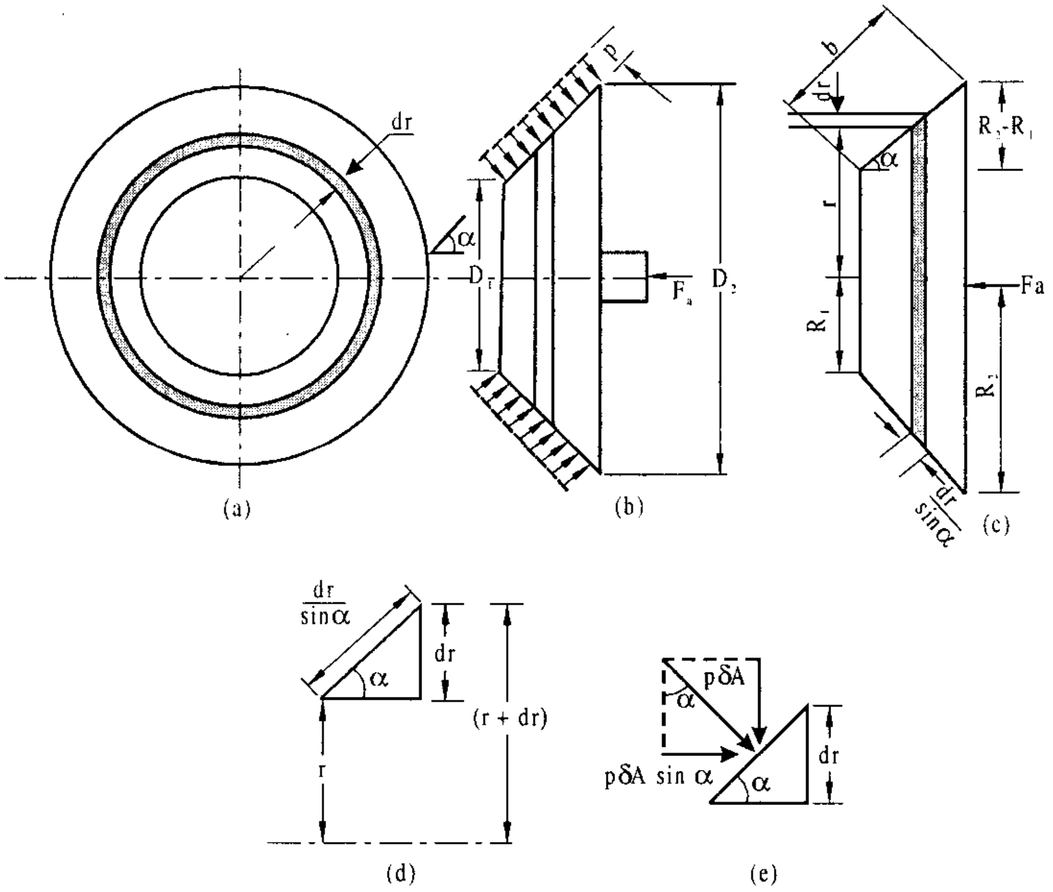


Fig. 6.5

Let d = diameter of shaft

D_1 = Inner diameter of cone; R_1 = Inner radius of cone

D_2 = Outer diameter of cone; R_2 = Outer radius of cone

D_m = Mean diameter of cone; R_m = Mean radius of cone

α = Half cone angle or Semi cone angle or Face angle or Pitch cone angle

2α = Cone angle

b = Face width or width of cone

F_a = Axial force

F_n = Normal force = $\dfrac{F_a}{\sin\alpha}$

M_t = Torque transmitted

N = Power in kW

n = Speed in rpm

p = Intensity of normal pressure in N/mm^2

μ = Coefficient of friction.

Consider an elemental frustum of cone bounded by circles of radii r and (r + dr). For this elemental frustum:

$$\text{Area } \delta_A = 2\pi r \times \text{sloping length} = 2\pi r \times \left(\frac{dr}{\sin\alpha}\right)$$

$$\text{Normal force on the elemental frustum} = \delta_A \times p = 2\pi r\left(\frac{dr}{\sin\alpha}\right)\times p$$

$$\text{Axial force on the elemental frustum} = p\,\delta_A \sin\alpha = p \times 2\pi r\left(\frac{dr}{\sin\alpha}\right)\times \sin\alpha = p.2\pi r dr$$

$$\text{Total axial force } F_a = \int_{R_1}^{R_2} p\,2\pi r\,dr \qquad \text{---- (i)}$$

$$\text{Friction force on the elemental frustum} = \mu \times 2\pi r\left(\frac{dr}{\sin\alpha}\right)\times p$$

$$\text{Moment of friction force about the axis or friction torque} = \mu \times 2\pi r\left(\frac{dr}{\sin\alpha}\right)\times p \times r$$

$$\therefore \text{ Total torque } M_t = \int_{R_1}^{R_2} 2\pi\mu p r^2 \frac{dr}{\sin\alpha} \qquad \text{---- (ii)}$$

i) Uniform pressure theory

i.e., $p = \text{constant}$

$\therefore$ From equation (i) $F_a = 2\pi p \int_{R_1}^{R_2} r\,dr = \pi p (R_2^2 - R_1^2) = \dfrac{\pi p(D_2^2 - D_1^2)}{4}$ ---- 19.86 (b) (DDHB)

From equation (ii) $M_t = \dfrac{2\pi\mu p}{\sin\alpha} \int_{R_1}^{R_2} r^2 dr = \dfrac{2\pi\mu p}{\sin\alpha}\left(\dfrac{R_2^3 - R_1^3}{3}\right)$

$= \dfrac{2\pi\mu}{\sin\alpha} \cdot \dfrac{F_a}{\pi(R_2^2 - R_1^2)} \cdot \left(\dfrac{R_2^3 - R_1^3}{3}\right)$ $[\because F_a = \pi p (R_2^2 - R_1^2)]$

$= \dfrac{2\mu F_a}{3\sin\alpha} \cdot \left(\dfrac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right) = \dfrac{\mu F_a}{2\sin\alpha} \times \dfrac{2}{3}\left(\dfrac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right)$

$= \dfrac{\mu F_a D_m}{2\sin\alpha}$ ---- 19.78 (DDHB)

where mean diameter $D_m = \dfrac{2}{3}\left(\dfrac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right)$ ---- 19.85 a (DDHB)

ii) Uniform wear theory

i.e., $pr = \text{constant} = c$

$\therefore$ From equation (i) $F_a = 2\pi c \int_{R_1}^{R_2} dr = 2\pi c (R_2 - R_1)$

From equation (ii) $M_t = \dfrac{2\pi\mu c}{\sin\alpha} \int_{R_1}^{R_2} r\,dr = \dfrac{\pi\mu c}{\sin\alpha} (R_2^2 - R_1^2)$

$= \dfrac{\pi\mu}{\sin\alpha} \cdot \dfrac{F_a}{2\pi(R_2 - R_1)} \cdot (R_2^2 - R_1^2)$ $\{\because F_a = 2\pi c (R_2 - R_1)\}$

$= \dfrac{\mu F_a}{2\sin\alpha} (R_2 + R_1) = \dfrac{\mu F_a R_m}{\sin\alpha}$

$= \dfrac{\mu F_a D_m}{2\sin\alpha}$ ---- 19.78 (DDHB)

where mean diameter $D_m = \dfrac{D_2 + D_1}{2}$

Axial force $F_a = 2\pi c (R_2 - R_1) = 2\pi\, pr (R_2 - R_1)$

Considering pressure at the mean radius, $F_a = 2\pi p R_m (R_2 - R_1) = 2\pi p \dfrac{D_m}{2}\left(\dfrac{D_2 - D_1}{2}\right)$

$$= \pi p D_m \left(\frac{D_2 - D_1}{2}\right) = \pi p D_m \left(\frac{b \times 2\sin\alpha}{2}\right)$$

$\therefore\ F_a = \pi D_m pb \sin\alpha$ ---- 19.75 (DDHB)

where face width $b = \dfrac{D_2 - D_1}{2\sin\alpha}$ (Refer figure 4.5 c)

Outer diameter $D_2 = D_m + b\sin\alpha$

Inner diameter $D_1 = D_m + b\sin\alpha.$

The force necessary to engage the clutch when one member is rotating

$F_a' = F_n (\sin\alpha + \mu\cos\alpha)$ ---- 19.80 (DDHB)

6.10 PROCEDURAL STEPS FOR CONE CLUTCH

Type I Mean diameter D_m is unknown

i) Torque transmitted

$M_t = 9550 \times 1000 \times \dfrac{N}{n}$ where M_t in Nmm ---- 19.3 c (DDHB)

ii) Diameter of shaft

$d = \sqrt[3]{\dfrac{16M_t}{\pi\tau_s\eta}}$ ---- 19.49 (DDHB)

iii) Mean diameter

$q = \dfrac{D_m}{b} = 4.5$ to 8 ---- 19.81 (DDHB)

$\therefore$ Choose $\dfrac{D_m}{b} = 6$

Express b in terms of D_m

iv) Axial force

$F_a = \pi D_m pb \sin\alpha$ ---- 19.75 (DDHB)

Express F_a in terms of D_m

v) Dimensions

Also torque transmitted $M_t = \dfrac{\mu F_a D_m}{2\sin\alpha}$ ---- 19.78 (DDHB)

$\therefore$ Mean diameter D_m

Face width b

Inner diameter D_1
Outer diameter D_2
Axial force F_a

Type II D_m is known

i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n}$$

ii) Diameter of shaft

$$d = \sqrt[3]{\frac{16M_t}{\pi\tau_s\eta}}$$

iii) Axial force

$$M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha} \quad \therefore \text{ Axial force } F_a$$

iv) Dimensions

$$F_a = \pi D_m pb \sin\alpha$$

∴ face width b; inner diameter D_1; outer diameter D_2

Example 6.9

Design the main dimensions of a cone clutch to transmit 40 kW at 1000 rpm. Assume suitable material for friction lining.

VTU January/February 2004

Data :

N = 40 kW; n = 1000 rpm

Solution :

Assume leather as the friction material. From Table 19.7 for leather $\mu = 0.3$ to 0.5;

p = 0.0686 to 0.2746 MPa; adopt $\mu = 0.4$ and p = 0.2 N/mm². Assume uniform wear criterion

i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{40}{1000} = 382000 \text{ Nmm}$$

ii) Mean diameter

$$\frac{D_m}{b} = 4.5 \text{ to } 8 \qquad \text{---- 19.81 (DDHB)}$$

Take $\frac{D_m}{b} = 6 \quad \therefore \quad b = \frac{D_m}{6}$

iii) Axial force

$$F_a = \pi D_m pb \sin\alpha \quad \text{---- 19.75 (DDHB)}$$

For leather $\alpha = 12.5°$

$$\therefore \quad F_a = \pi D_m \times 0.2 \times \frac{D_m}{6} \times \sin 12.5 = 0.0227\ D_m^2$$

iv) Dimensions

$$\text{Also } M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha} \quad \text{---- 19.78 (DDHB)}$$

$$\text{i.e., } 382000 = \frac{1}{2} \times \frac{0.4 \times 0.0227 D_m^2 \times D_m}{\sin 12.5}$$

$\therefore$ Mean diameter $D_m = 263.23 \text{ mm} \approx 264 \text{ mm}$

Face width $b = \frac{D_m}{6} = \frac{264}{6} = 44 \text{ mm}$

Inner diameter $D_1 = D_m - b\sin\alpha = 264 - 44 \sin 12.5 = 254.5 \text{ mm}$

Outer diameter $D_2 = D_m + b\sin\alpha = 264 + 44 \sin 12.5 = 273.5 \text{ mm}$

Axial force $F_a = 0.0227\ D_m^2 = 0.0227 \times 264^2 = 1579.7 \text{ N}$

Example 6.10

A cone clutch has a semi-cone angle of 12° to transmit 10 kW at 750 rpm. The width of the face is one fourth of the mean diameter of friction lining. If the normal intensity of pressure between the contacting surface is not to exceed 0.85 bar, assuming uniform wear criterion and taking μ = 0.2. Calculate dimensions of clutch. Also find the axial force while running i.e., at the beginning of engagement.

VTU August 2001 ; Dec'07/Jan'08 ; Dec'08/Jan'09

Data :

$\alpha = 12°$; $N = 10$ kW; $n = 750$ rpm; $b = \frac{1}{4} D_m$; $p = 0.85 \text{ bar} = 0.85 \times \frac{10^5}{10^6} = 0.085 \text{ N/mm}^2$

$\mu = 0.2$; **uniform wear criterion.**

Solution :

i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{10}{750} = 127333.33 \text{ Nmm}$$

ii) Axial force

$$F_a = \pi D_m pb \sin\alpha = \pi D_m \times 0.085 \times \left(\frac{D_m}{4}\right) \sin 12 = 0.01388\ D_m^2$$

iii) Dimensions

$$\text{Also } M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha}$$

$$\text{i.e.,}\quad 127333.33 = \frac{1}{2} \times \frac{0.2 \times 0.01388 D_m^2 \times D_m}{\sin 12}$$

$\therefore$ Mean diameter $D_m = 267.2 \text{ mm} \approx 268 \text{ mm}$

Face width $b = \frac{D_m}{4} = \frac{268}{4} = 67 \text{ mm}$

Inner diameter $D_1 = D_m - b \sin \alpha = 268 - 67 \sin 12 = 254 \text{ mm}$

Outer diameter $D_2 = D_m + b \sin \alpha = 268 + 67 \sin 12 = 282 \text{ mm}$

Axial load $F_a = 0.01388\, D_m^2 = 0.01388 \times 268^2 = 996.9 \text{ N}$

iv) Axial force required when one member is rotating

$F'_a = F_n (\sin \alpha + \mu \cos \alpha)$ ---- 19.80 (DDHB)

where $F_n = \frac{F_a}{\sin \alpha} = \frac{996.9}{\sin 12} = 4794.824 \text{ N}$ ---- 19.76 (DDHB)

$\therefore\ F'_a = 4794.824\, [\sin 12 + 0.2 \cos 12] = 1934.91 \text{ N}$

Example 6.11

A friction cone clutch has to transmit a torque of 200 Nm at 1440 rev/min. The larger diameter of the cone is 350 mm, the cone pitch angle is 6.25°. The face width is 65 mm. The coefficient of friction is 0.2. Determine.

(a) The axial force required to transmit the torque.

(b) The average normal pressure on the contact surfaces when the maximum torque is transmitted.

VTU July/August 2004

Data :

$M_t = 200 \text{ Nm} = 2 \times 10^5 \text{ Nmm}$; $n = 1440$ rpm; $D_2 = 350$ mm; $\alpha = 6.25°$; $b = 65$ mm; $\mu = 0.2$

Solution :

a) Axial force

Outer diameter $D_2 = D_m + b \sin \alpha$

i.e., $350 = D_m + 65 \times \sin 6.25$

$\therefore$ Mean diameter $D_m = 342.92 \text{ mm}$

Torque transmitted $M_t = \frac{1}{2} \times \frac{0.2 \times F_a \times 342.92}{\sin 6.25}$

$\therefore$ Axial force required $F_a = 634.934 \text{ N}$

b) Average normal pressure

Axial force $F_a = \pi D_m\, p\, b \sin \alpha$

i.e., $634.934 = \pi \times 342.92 \times p \times 65 \times \sin 6.25$

$\therefore$ Average normal pressure $p = 0.0833 \text{ N/mm}^2$

Example 6.12

An engine developing 30 kW at 1250 rpm is fitted with a cone clutch. The cone has a face angle of 12.5°. The mean diameter is 400 mm, μ = 0.3 and the normal pressure is not to exceed 0.08 N/mm². Design the clutch.

Data :

$N = 30$ kW; $n = 1250$ rpm; $\alpha = 12.5°$; $D_m = 400$ mm; $\mu = 0.3$; $p = 0.08$ N/mm²

Solution :

i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{30}{1250} = 229200 \text{ Nmm}$$

ii) Axial force

$$\text{Also } M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha} \qquad \text{---- 19.78 (DDHB)}$$

$$\text{i.e., } 229200 = \frac{1}{2} \times \frac{0.3 \times F_a \times 400}{\sin 12.5}$$

$$\therefore \text{ Axial force } F_a = 826.8 \text{ N}$$

iii) Dimensions

$$\text{Also } F_a = \pi D_m p\, b \sin\alpha \qquad \text{---- 19.75 (DDHB)}$$

$$\text{i.e., } 826.8 = \pi \times 400 \times 0.08 \times b \times \sin 12.5$$

$$\therefore \text{ Width of face } b = 38 \text{ mm}$$

$$\text{Inner diameter } D_1 = D_m - b\sin\alpha = 400 - 38 \sin 12.5 = 392 \text{ mm}$$

$$\text{Outer diameter } D_2 = D_m + b\sin\alpha = 400 + 38 \sin 12.5 = 408 \text{ mm}$$

Example 6.13

Compare the power capacity of two clutches (i) a multiple disc clutch and (ii) a cone clutch. Both clutches operate at the same speed. Both have the same mean diameter, same coefficient of friction and same axial load. The multiple disc clutch has 4 steel and 3 bronze disc. The total cone angle of the cone clutch is 20°. Assume uniform wear theory for both clutches. **(B.U. August 1995)**

Data :

$n_m = n_c$; $(D_m)_m = (D_m)_c$; $\mu_m = \mu_c$; $(F_a)_m = (F_a)_c$; $i_1 = 3$; $i_2 = 4$; $2\alpha = 20°$ $\therefore \alpha = 10°$; uniform wear theory

Solution :

Total number of active surface $i = i_1 + i_2 - 1 = 3 + 4 - 1 = 6$

$$\text{Power } N = \frac{M_t \times n}{9550} \quad \text{where } M_t \text{ in Nm}$$

Since speed is constant $N \propto M_t$

$$\therefore \frac{\text{Torque transmitted by multiplate clutch}}{\text{Torque transmitted by cone clutch}} = \frac{(M_t)_m}{(M_t)_c} = \frac{\left(\frac{1}{2}\mu F_a D_m i\right)_m}{\left(\frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha}\right)_c} = i.\sin\alpha = 6\sin 10 = 1.042$$

$$\therefore \frac{\text{Power capacity of multiple disc clutch}}{\text{Power capacity of cone clutch}} = 1.04$$

Example 6.14

Design a cone clutch to transmit a power of 40kW at a rated speed of 750 rpm. Also determine

(i) Axial force necessary to transmit torque.

(ii) Axial force necessary to engage the cone clutch.

(VTU Jan/Feb 2005, Jan/Feb 2006)

Data:

N = 40 kW; n = 750 rpm

Solution :

Assume leather as the friction material. From Table 19.7 for leather μ = 0.3 to 0.5; p = 0.0686 MP*a* to 0.2746 MP*a*. Take μ = 0.4 and p = 0.2 N/mm²

Assume uniform wear criterion.

(a) Design of cone clitch

(i) Torque transmitted

$$M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{40}{750} = 509333.33\text{Nmm}$$

(ii) Mean diameter

$$\frac{D_m}{b} = 4.5 \text{ to } 8 \qquad \text{---- 19.81 (DDHB)}$$

$$\text{Take } \frac{D_m}{b} = 6. \therefore b = \frac{D_m}{6}$$

(iii) Axial force

$$F_a = \pi D_m pb \sin\alpha \qquad \text{---- 19.75 (DDHB)}$$

$$\text{For leather } \alpha = 12.5°$$

$$\therefore F_a = \pi D_m \times 0.2 \times \frac{D_m}{6} \times \sin 12.5° = 0.0227\ D_m^2$$

(iv) Dimensions

$$\text{Also } M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha} \qquad \text{---- 19.78 (DDHB)}$$

$$\text{i.e., } 509333.33 = \frac{1}{2} \times \frac{0.4 \times 0.0227 D_m^2 \times D_m}{\sin 12.5°}$$

$\therefore$ Mean diameter D_m = 289.6mm≅290mm

Face width b = $\frac{D_m}{6} = \frac{290}{6} = 48.3$ mm

Inner diameter D_1 = $D_m - b \sin\alpha = 290 - 48.3 \times \sin 12.5 = 279.5$ mm

outer diameter D_2 = $D_m + b \sin\alpha = 290 + 48.3 \times \sin 12.5 = 300.5$ mm

(b) Axial force necessary to transmit torque

We have $M_t = \frac{1}{2}\frac{\mu F_a D_m}{\sin\alpha}$

i.e., $509333.33 = \frac{1}{2} \times \frac{0.4 \times F_a \times 290}{\sin 12.5}$

$\therefore$ Axial force necessary to transmit the torque F_a = 1900. 7N.

(c) Axial force necessary to engage the cone clutch.

Axial force required when one member is rotating

$F_a = F_n [\sin\alpha + \mu \cos\alpha]$ ---- 19.80 (DDHB)

where $F_n = \frac{F_a}{\sin\alpha} = \frac{1900.7}{\sin 12.5} = 8781.66$ N ---- 19.76 (DDHB)

$\therefore$ $F_a = 8781.66 [\sin 12.5 + 0.4 \cos 12.5] = 5330.1$N

i.e., Axial force necessary to engage the cone clutch F_a = 5330.1N

6.11 CENTRIFUGAL CLUTCH

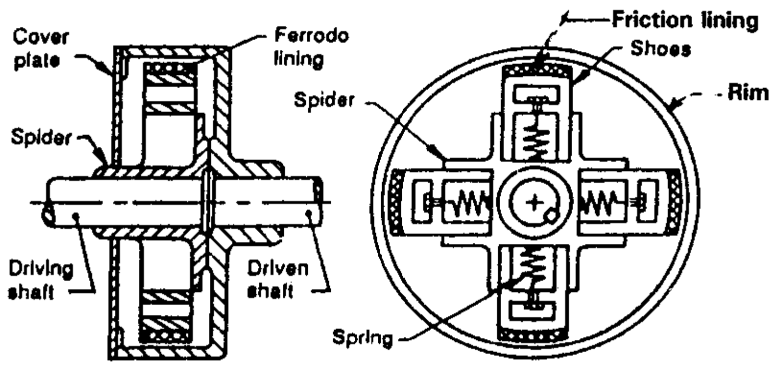


Fig. 6.6

Figure 6.6 shows a centrifugal clutch. It mainly consists of a rim and a spider. The spider is connected to the driver or input shaft and the rim is connected to the driven or output shaft. The spider normally carries four shoes and the outer faces of the shoes are lined with friction material. When the driver starts rotating, the masses moves radially outward due to centrifugal force and press against the rim which is connected to the output shaft. The springs exert a radially inward force which is assumed constant. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. If the centrifugal force is less than the spring force then the shoes remains in the same position (i.e., when the driving shaft is stationary) The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

6.12 TORQUE TRANSMITTED BY CENTRIFUGAL CLUTCH

Let m = Mass of each shoe

w = Weight of each shoe = mg

i = Number of shoes

r' = Inner radius of drum

r = Distance between cg of the shoe and shaft axis

l = Length of shoe

b = Width of shoe

n_1 = Speed at which engagement commences in rpm

n_2 = Running speed in rpm

ω_1 = Angular velocity of engagement speed = $\frac{2\pi n_1}{60}\ \frac{\text{rad}}{\text{sec}}$

ω_2 = Angular velocity of running speed = $\frac{2\pi n_2}{60}\ \frac{\text{rad}}{\text{sec}}$

F_{C_1} = Centrifugal force at engagement speed $= \frac{w}{g}\ \omega_1^2\ r$ ---- 19.114

F_{C_2} = Centrifugal force at running speed $= \frac{w}{g}\ \omega_2^2\ r$ ---- 19.115

F_C = Outward radial force on the rim = $F_{C_2} - F_{C_1}$ ---- 19.116 a

$= \frac{w}{g}\left(\omega_2^2 - \omega_1^2\right) r$ ---- 19.116 b

μ = Coefficient of friction

η = Keyway factor

p = Allowable pressure

θ = Angle subtended by the shoe at the centre

M_t = Torque transmitted

N = Power in kW

S = Spring force

k = Stiffness of spring

c = Radial clearance between block and drums

Net force acting on the drum $F_c = \frac{w}{g}\left(\omega_2^2 - \omega_1^2\right) r$

Friction force = $\mu F_c = \mu \frac{w}{g}\left(\omega_2^2 - \omega_1^2\right) r$

Frictional torque per shoe = $\mu F_c r' = \mu \frac{w}{g}\left(\omega_2^2 - \omega_1^2\right) r.r'$

$\therefore$ Total frictional torque $M_t = i\,\mu \frac{w}{g}\left(\omega_2^2 - \omega_1^2\right) r\, r' = i\mu F_c.r'$ ---- 19.118 (DDHB)

Weight and size of shoes

Also $M_t = 9550 \times 1000 \times \frac{N}{n}$ ---- 19.3 c (DDHB)

Using equations 19.3 c and 19.118 the weight of shoes can be calculated

Length of shoe $l = r'\theta$ (refer figure 4.7)

Area of the shoe = lb

$\therefore$ Net force on the shoe F_c = Area × pressure = lbp

$\therefore$ Width of shoe $b = \frac{F_c}{lp}$ ---- 19.119

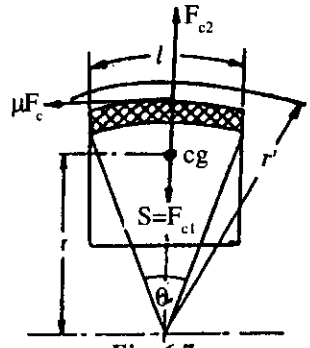


Fig. 6.7

Example 6.15

Design a centrifugal clutch for the following details :

Power transmitted = 12 kW
Speed = 800 rpm
Inner radius of pulley = 130 mm
Number of shoes = 4
Coefficient of friction = 0.25
Intensity of normal pressure = 0.2 N/mm²
Engagement speed = 75% of the running speed

Distance between cg of the shoe and shaft axis = 110 mm

Data :

$N = 12$ kW; $n_2 = 800$ rpm; $r' = 130$ mm; $i = 4$; $\mu = 0.25$; $p = 0.2$ N/mm²; $n_1 = \frac{3}{4} \times 800 = 600$ rpm;

$r = 110$ mm

Solution :

i) **Torque transmitted**

$$M_t = 9550 \times 1000 \times \frac{N}{n_2} = 9550 \times 1000 \times \frac{12}{800} = 143250\,\text{N mm}$$

ii) Mass or weight of each shoe

Angular velocity at rated speed $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi 800}{60} = 83.7758 \frac{\text{rad}}{\text{sec}}$

Angular velocity at engagement speed $\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi 600}{60} = 62.8319 \frac{\text{rad}}{\text{sec}}$.

Outward radial force $F_c = F_{c2} - F_{c1} = \frac{w}{g}\left(\omega_2^2 - \omega_2^2\right) r$ ---- 19.116 b (DDHB)

$= \frac{w}{9810} (83.7758^2 - 62.8319^2)\ 110 = 34.43\ w$

Now, Total frictional torque $M_t = i\mu F_c r'$ ---- 19.118 (DDHB)

i.e., $143250 = 4 \times 0.25 \times 34.43\ w \times 130$

$\therefore$ Weight of each shoe $w = 32$ N

iii) Size of shoe

Length of shoe $l = r'\theta$ Assume $\theta = 60° = \frac{\pi}{3}$ radian

$= 130 \times \frac{\pi}{3} = 136.14$ mm

Width of shoe $b = \frac{F_c}{l.p}$ ---- 19.119 (DDHB)

$= \frac{34.43w}{l.p} = \frac{34.43 \times 34}{136.14 \times 0.2} = 43$ mm

---

Note :

If μ and p are not given then assume leather as the friction material and select the values from Table 19.7.

Example 6.16

A centrifugal clutch has 4 shoes. When the clutch is at rest, each shoe is pulled against a stop by a spring so as to leave a radial clearance of 5 mm between the shoe and the rim. The pull exerted by the spring is then 500 N. The mass centre of the shoe is 160 mm from the axis of the clutch. If the internal diameter of the rim is 400 mm, the mass of each shoe is 8 kg, the stiffness of spring is 50 N/mm and the coefficient of friction between the shoe and the rim is 0.3, find the power transmitted by the clutch at 500 rpm.

Data :

c = 5 mm; i = 4; S = 500 N; r = 160 mm; d' = 400 mm $\therefore$ r' = 200 mm; m = 8 kg;
k = 50 N/mm; μ = 0.3; n_2 = 500 rpm

Solution :

i) Torque transmitted

Since clearance is given the distance between cg of shoe and shaft axis

$r_1 = r + c = 160 + 5 = 165 \text{ mm} = 0.165 \text{ m}$

Radially inward spring force $F_{c1} = S + kc = 500 + 50 \times 5 = 750 \text{ N}$

Radially outward centrifugal force $F_{c2} = m\omega_2^2 r_1 = m\left(\frac{2\pi n_2}{60}\right)^2 r_1$

$$= (8)\left(\frac{2\pi 500}{60}\right)^2 (0.165) = 3618.855 \text{ N}$$

$\therefore$ Net outward force $F_c = F_{c2} - F_{c1} = 3618.855 - 750 = 2868.855 \text{ N}$

Total frictional torque $M_t = i\mu F_c r'$ ---- 19.118 (DDHB)

$$= 4 \times 0.3 \times 2868.855 \times 200 = 688525.2 \text{ Nmm}$$

Also $M_t = 9550 \times 1000 \times \frac{N}{n_2}$ where M_t in Nmm

i.e., $688525.2 = 9550 \times 1000 \times \frac{N}{500}$

$\therefore$ Power transmitted $N = 36.05 \text{ kW}$

Example 6.17

A plate clutch with a maximum diameter of 600 mm has maximum lining pressure of 0.35 MPa. The power to be transmitted at 400 rpm is 135 kW and μ = 0.3. Find inside diameter and spring force required to engage the clutch, if the springs with spring index 6 and material of spring is steel with safe shear stress 600 MPa is used. Find the wire diameter if 6 springs are used.

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Data :

Clutch	**Spring**
$D_2 = 600$ mm; $p = 0.35$ N/mm²	$c = 6$; $\tau = 600$ MPa
$n = 400$ rpm; $N = 135$ kW	Number of springs = 6
$\mu = 0.3$	$d = ?$
$D_1 = ?$ $F_a = ?$	

Solution :

a) Design of clutch

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{135}{400} = 3223125 \text{ Nmm}$

Assume uniform wear condition.

Axial force $F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1)$ ---- 19.83 (DDHB)

$$= \frac{1}{2}\pi \times 0.35 D_1 (600 - D_1)$$

$$= 0.55 D_1 (600 - D_1)$$

Mean diameter $D_m = \frac{1}{2}(D_2 + D_i) = \frac{1}{2}(600 + D_i)$ ---- 19.85 b (DDHB)

Assume both sides of the friction plate is effective $\therefore$ i = 2

$\therefore$ Torque transmitted $M_t = \frac{1}{2}\mu F_a D_m . i$

i.e., $3223125 = \frac{1}{2} \times 0.3 \times 0.55\, D_i (600 - D_i) \times \frac{1}{2}(600 + D_i) \times 2$

$39083906.74 = 360000\, D_i - D_i^3$

i.e., $360000\, D_i - D_i^3 - 39083906.74 = 0$

By trial and error method

$D_i = 535$ mm = inside diameter of friction surface

$\therefore$ Axial force $F_a = 0.55\, D_i (600 - D_i) = 0.55 \times 535\, [600 - 535]$

$= 19126.25$ N

b) Design of spring

Force on each spring $F = \frac{\text{Total axial force}}{\text{Number of springs}} = \frac{19126.25}{6} = 3187.708$ N

Spring index $c = 6 = \frac{D}{d}$ $\therefore$ $D = 6d$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$

Shear stress $\tau = \frac{8FDk}{\pi d^3}$ ---- 20.22 (DDHB)

i.e., $600 = \frac{8 \times 3187.708 \times 6d \times 1.2525}{\pi d^3}$

$d = 10.08$ mm

From Table 20.12 (Old DDHB) standard wire diameter d = 10.5 mm

6.13 BRAKES

A brake is defined as a mechanical device used to control the motion by absorbing kinetic energy of a moving body or by absorbing potential energy of the objects being lowered by elevators, hoists, etc. The energy absorbed by brakes is dissipated in the form of heat. The materials commonly used for facing or lining of brakes are wood, leather, asbestos and Ferrodo. The capacity of any brake depends upon the unit pressure between the braking surfaces, the coefficient of friction and the heat radiating capacity of the brake.

The brakes may be classified as pneumatic, hydraulic, electric and mechanical brakes. This chapter deals with only mechanical brakes. The following are the important types of mechanical brakes.

(i) Block or shoe brake (ii) Band brake (iii) Band and block brake (iv) Internal expanding shoe brake.

6.14 ENERGY TO BE DISSIPATED

The first step in the design of a mechanical brake is to determine the braking-torque capacity for the given application. The braking-torque depends upon the amount of energy absorbed by the brake. When a mechanical system of mass m, moving with velocity v_1 is slowed down to velocity v_2 then, the decrease of kinetic energy for a change of speed v_1 to v_2,

$$E_k = \frac{F(v_1^2 - v_2^2)}{2g} \quad \text{---- 19.135 a (DDHB)}$$

where F = Load of moving part

v_1, v_2 = Speed of live load before and after brake.

The change of kinetic energy of all rotating parts such as hoist drum and various gears etc.

$$E_r = \frac{\Sigma W k_0^2(\omega_1^2 - \omega_2^2)}{2g} \quad \text{---- 19.136 (DDHB)}$$

where ω_1, ω_2 = angular velocity of the rotating parts

k_0 = radius of gyration of these parts.

If the potential energy of the moving part also changes during braking period, then the change in potential energy has also to be added to the change in kinetic energy to obtain the total energy to be dissipated.

Change in potential energy $E_p = F(h_1 - h_2) = mg(h_1 - h_2)$ or

Change in potential energy absorbed by the brake during the time t,

$$E_p = \frac{F}{2}(v_1 + v_2)\,t \quad \text{---- 19.135 (DDHB)}$$

∴ Total energy to be dissipated = $E_k + E_r + E_p$

Frictional work done in time t, $W_k = \dfrac{F_0 . \pi D(n_1 + n_2)t}{2 \times 60}$ ---- 19.137 (DDHB)

where F_0 = tangential force

Since total energy to be dissipated is equal to the work done

$$E_k + E_r + E_p = \frac{F_0 . \pi D(n_1 + n_2)t}{2 \times 60}$$

∴ Tangential force $F_0 = \dfrac{38.2(E_k + E_p + E_r)}{D(n_1 + n_2)t}$ ---- 19.138 (DDHB)

∴ Torque transmitted when the blocks are pressed against the drum $M_t = F_0 \dfrac{D}{2} = \mu F_n . \dfrac{D}{2}$

where F_n = Normal force; D = Diameter of brake drum

μ = Coefficient of friction

If $2\theta > 60°$ then equivalent coefficient of friction $\mu' = \dfrac{4\mu\sin\theta}{2\theta+\sin 2\theta}$

$$\therefore \quad M_t = \mu F_n \frac{D}{2}\left(\frac{4\sin\theta}{2\theta+\sin 2\theta}\right) \qquad \text{---- 19.141 (DDHB)}$$

6.15 HEATING OF BRAKES

If the brakes are used continuously then the temperature will increase until the heat generation becomes equal to the heat dissipation. The rate of heat generation depends upon the braking area, the radiating surface, unit pressure and the air circulation.

$\therefore$ Heat generated from work of friction $H_g = \mu\, p\, A_c\, v$, Joules/sec or watts ---- 19.191a (DDHB)

$= \mu F_n v \ (\because \ pA_c = F_n)$

$= F_\theta . v \ (\because \ \mu F_n = F_\theta)$

where A_c = Area

p = Unit pressure; v = velocity

μ = Coefficient of friction

F_n = Normal force

F_θ = Tangential force

For a brake lowering the load, the heat to be radiated $H = Wh$, Joules---- 19.192 a (DDHB)

where w = Weight lowered

h = Total distance in meters

The ability of the brake drum to absorb heat is proportional to the mass and to the specific heat of the material. Assuming that all heat generated is absorbed by the brake drum and its supporting flange, the temperature rise is given by $\Delta T = \dfrac{H}{mC}$ ---- 19.194 (DDHB)

where m = Mass of brake drum and flange in kg

C = Specific heat of material

= 500 J/kg° C for CI

= 456.5 J/kg° C for steel

Also heat generated, $H_g = 754\ k_l N$, Joules/sec ---- 19.193 a (DDHB)

where k_l = load factor or ratio of the actual brake operating time to the total cycle time of operation.

Now rate of heat dissipation, $H_d = C_2\, \Delta T A_r$, Joules/sec or watts ---- 19.195 a (DDHB)

where C_2 = radiating factor from Table 19.13

A_r = radiating surface

For $H_g = H_d$

$$A_r = \frac{754k_l N}{C_2 \Delta T} = \frac{H_g}{C_2 \Delta T} \quad \text{---- 19.196 a}$$

6.16 BLOCK OR SHOE BRAKE

A single block or shoe brake is shown in Fig. 6.8. It consists of a shoe which may be rigidly mounted or pivoted to a lever. The block is pressed against the rim of a revolving brake wheel. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is made of a softer material than the rim of the wheel. Force is applied at one end of a lever and the other end of the lever is pivoted on a fixed fulcrum O. This type of brake is commonly used in railway trains, tram cars and in hoisting machinery. When the brake is applied, the lever with the block can be considered as a free body in equilibrium under the action of the following forces.

i) Applied force F at the end of the lever.
ii) Normal reaction F_n between the wheel and shoe.
iii) Tangential force F_θ between the wheel and shoe.
iv) Pin reaction.

Let M_t = Torque in Nmm
N = Power in kW
n = Speed in rpm
2θ = Angle of contact surface of the block
μ = Coefficient of friction
l = Length of shoe
b' = Width of shoe
p = Allowable pressure
F = Applied force or operating force
F_θ = Tangential braking force
F_n = Normal force
D = Diameter of brake drum
r = Radius of brake drum or wheel
a = Distance between the fulcrum pin and the centre of shoe
b = Distance between the centre of shoe and the applied force F
c = Distance between the fulcrum pin and the line of action of tangential force F_θ

Consider the following three cases

Case (i) Line of action of tangential force F_θ passes through the fulcrum

a) Direction of F_θ is towards the fulcrum

Taking moments about O (Refer Fig. 6.8)

$$F(a+b) = F_n.a = \frac{F_\theta}{\mu}.a \quad [\because \; F_\theta = \mu F_n]$$

$$\therefore \quad F = F_\theta \cdot \frac{a}{\mu(a+b)} \qquad \text{---- 19.144}$$

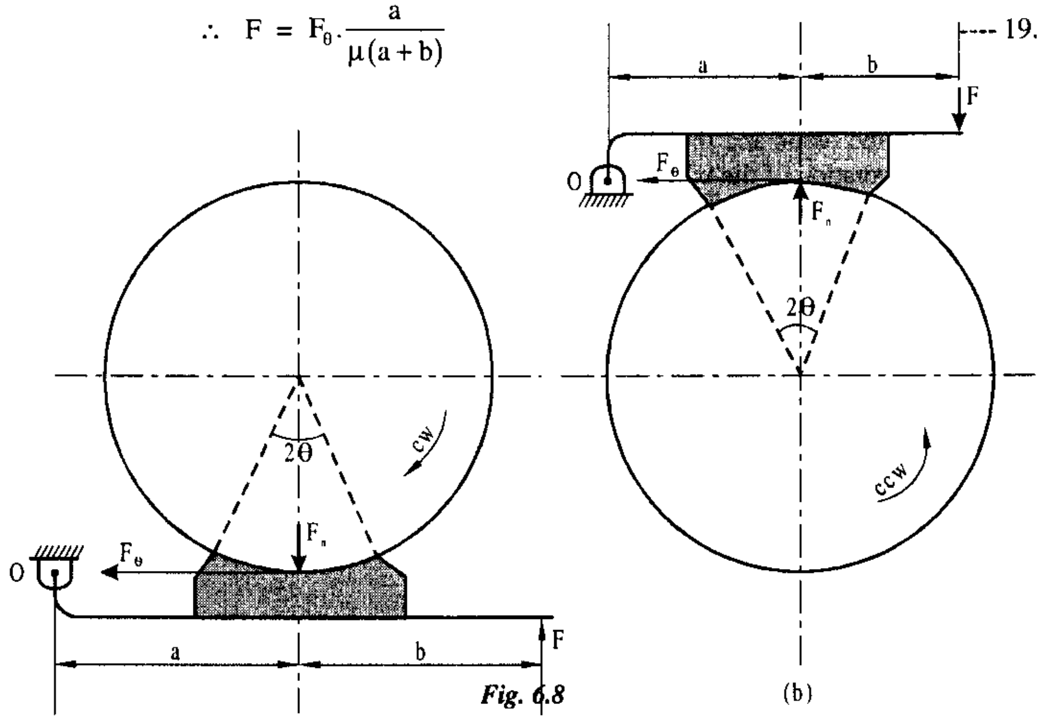


Fig. 6.8

b) Direction of F_θ is away from the fulcrum

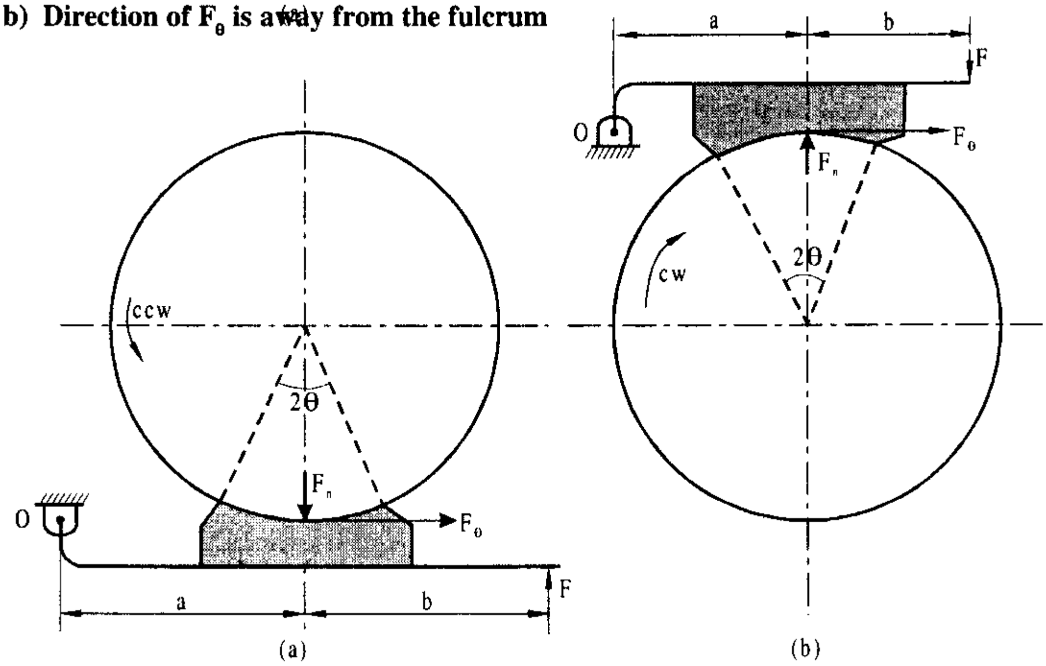


Fig. 6.9

Taking moments about O (Refer Fig. 6.9)

$$F(a+b) = F_n.a = \frac{F_\theta}{\mu}.a \quad \therefore \quad F = \frac{F_\theta.a}{\mu(a+b)} \qquad \text{---- 19.144}$$

Therefore if the line of action of tangential force F_θ passes through the fulcrum then the actuating force is the same whether the direction of F_θ is towards or away from the fulcrum.

Case (ii) Line of action of tangential force F_θ is in between the fulcrum and the center of drum

a) Direction of F_θ is towards the fulcrum

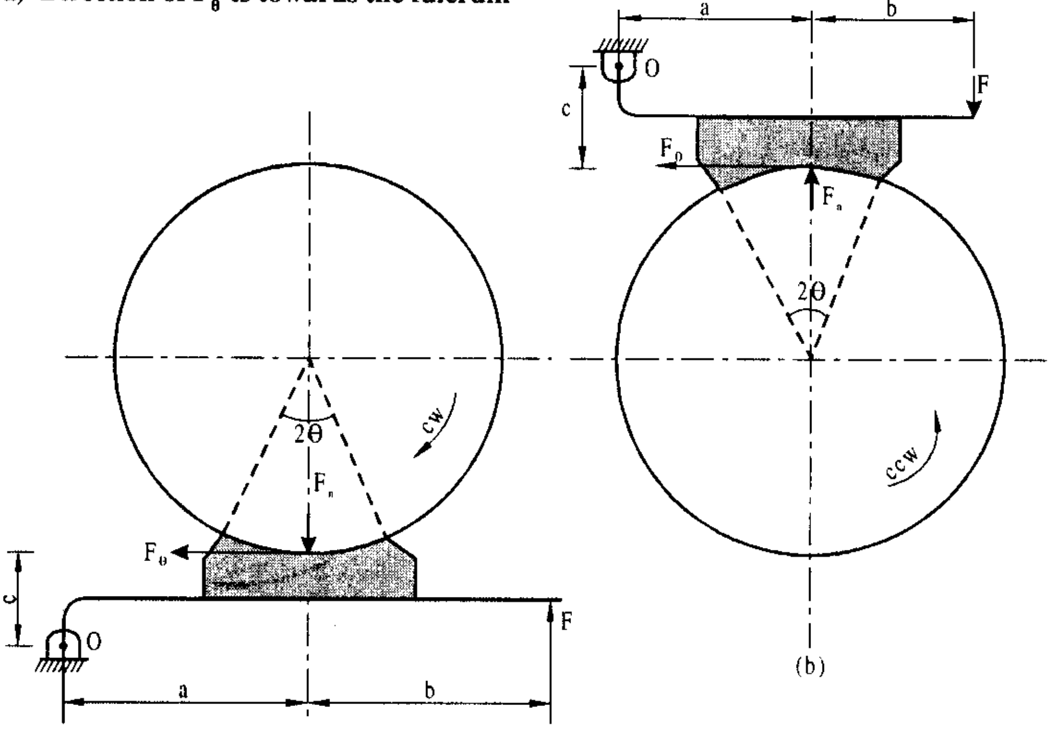


Fig. 6.10

Taking moments about O (Refer Fig. 6.10)

$$F(a+b) + F_\theta.c = F_n.a$$

$$\text{i.e.,} \quad F(a+b) = \frac{F_\theta}{\mu}a - F_\theta.c \quad (\because \ F_\theta = \mu F_n)$$

$$= F_\theta\left(\frac{a}{\mu} - c\right) = F_\theta.a\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

$$\therefore \ \text{Actuating force} \ F = \frac{F_\theta.a}{a+b}\left(\frac{1}{\mu} - \frac{c}{a}\right) \qquad \text{---- 19.145 (DDHB)}$$

b) Direction of F_θ is away from the fulcrum

Taking moments about O (Refer Fig. 6.11)

$$F(a+b) = F_n.a + F_\theta.c$$

$$= \frac{F_\theta}{\mu}.a + F_\theta.c = F_\theta.a\left[\frac{1}{\mu}+\frac{c}{a}\right]$$

$\therefore$ Actuating force $F = \dfrac{F_\theta.a}{a+b}\left(\dfrac{1}{\mu}+\dfrac{c}{a}\right)$ ---- 19.146 (DDHB)

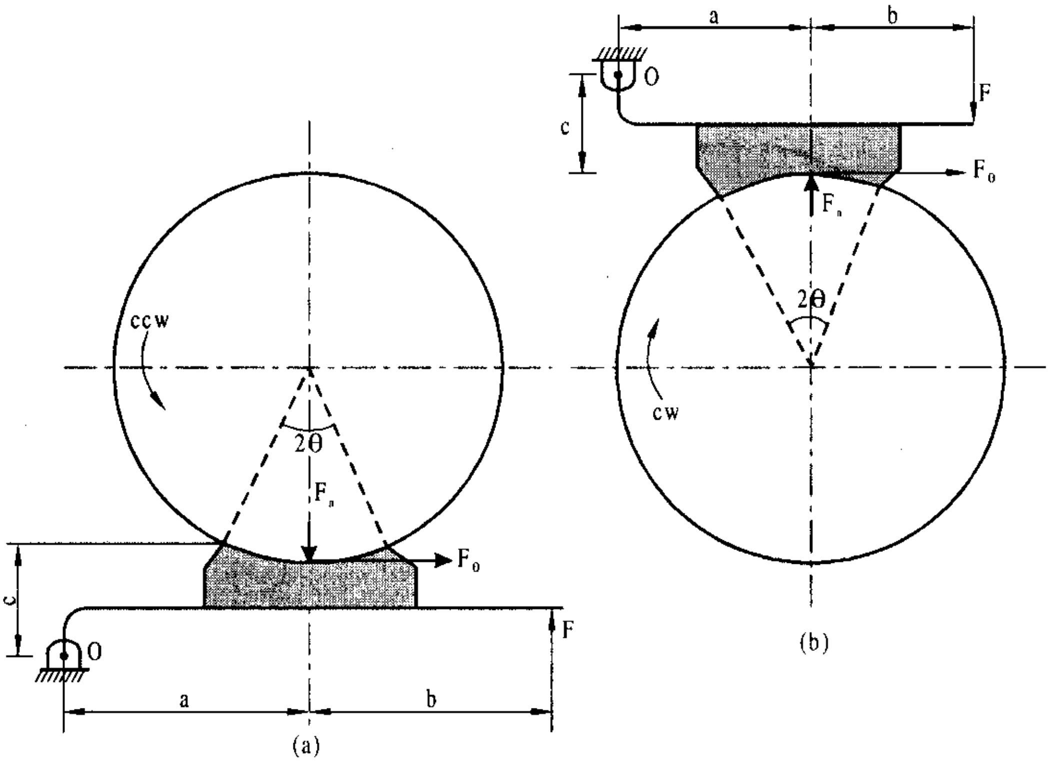


Fig. 6.11

Case (iii) Fulcrum is in between F_θ and centre of drum i.e., F_θ is either above or below the centre of drum and fulcrum.

a) Direction of F_θ is towards the fulcrum

Taking moments about O (Refer Fig. 6.12)

$$F(a+b) = F_n.a + F_\theta.c = \frac{F_\theta}{\mu}\cdot a + F_\theta.c = F_\theta.a\left(\frac{1}{\mu}+\frac{c}{a}\right)$$

$\therefore$ Actuating force $F = \frac{F_\theta . a}{a+b}\left(\frac{1}{\mu}+\frac{c}{a}\right)$ ---- 19.147 (DDHB)

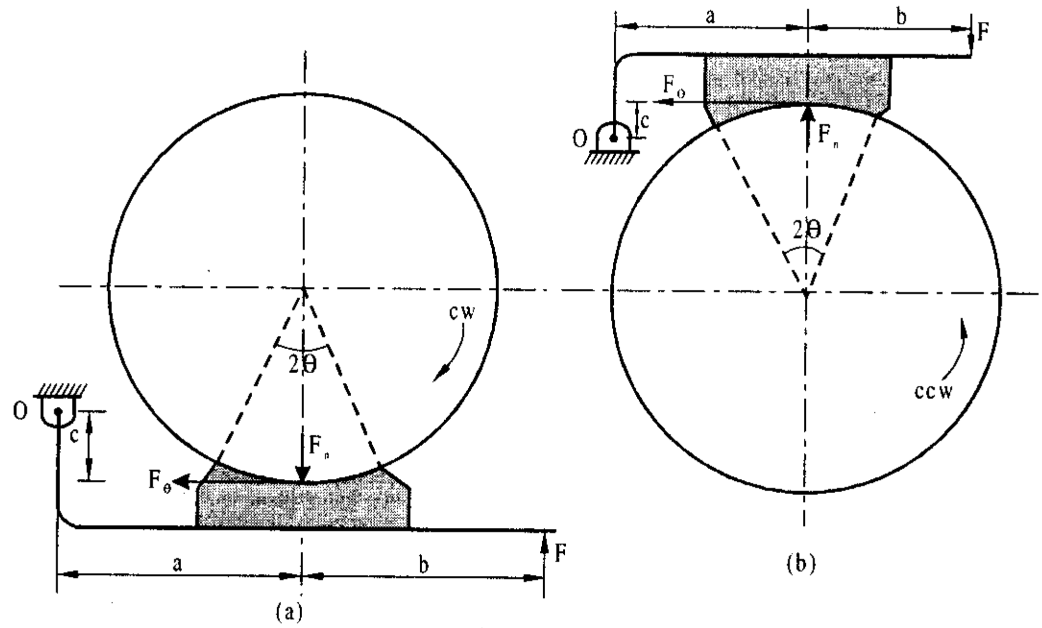


Fig. 6.12

b) Direction of F_θ is away from the fulcrum

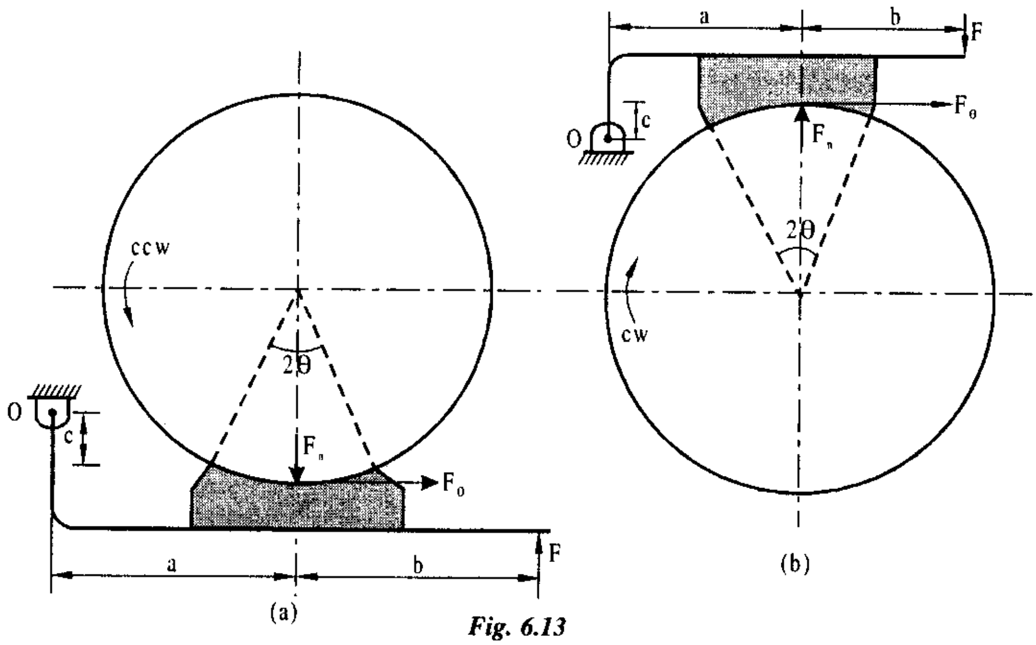


Fig. 6.13

Taking moments about O (Refer Fig. 6.13)

$$F(a+b) + F_\theta . c = F_n . a$$

$$= \frac{F_\theta}{\mu} \cdot a$$

i.e., $$F(a+b) = \frac{F_0}{\mu} \cdot a - F_0 . c = F_0 . a \left(\frac{1}{\mu} - \frac{c}{a} \right)$$

$\therefore$ Actuating force $$F = \frac{F_0 . a}{a+b} \left(\frac{1}{\mu} - \frac{c}{a} \right) \quad \text{---- 19.148 (DDHB)}$$

If the angle of contact 2 θ is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. When the angle of contact 2 θ is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases if the block or shoe is pivoted to the lever instead of being rigidly attached to the lever, the uniform wear of the brake lining will be in the direction of the applied force. Moreover if 2θ > 60°, then instead of actual coefficient of friction, equivalent coefficient of friction is used. Equivalent coefficient of friction $\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$ where, μ = Actual coefficient of friction.

If the frictional force helps to apply the brake, then the brake is said to be a self-energizing brake. If the frictional force is enough to apply the brake without any external force, then the brake is said to be a self-locking brake. The brake should be self-energizing and not the self-locking.

If p is the normal bearing pressure on the shoe then

$$p = \frac{F_n}{\text{Projected area of shoe}} = \frac{F_n}{2b' r \sin\theta}$$ where b' = Width of shoe

6.17 DESIGN CONSIDERATIONS FOR BLOCK BRAKE

i) Select the brake drum diameter for the given load, speed and other data.

ii) Determine the frictional force.

iii) Calculate the torsional moment capacity of the brake.

iv) Select the friction material and the corresponding coefficient of friction.

v) Find the normal reaction between the drum and the block or shoe.

vi) Calculate the necessary brake shoe area.

vii) Check for heat generation and heat dissipation.

Note :

Use clockwise rotation formula if F_θ is towards the fulcrum and use counter clockwise formula if F_0 is away from the fulcrum, from Table 19.1 (DDHB)

Example 6.18

A single block brake is shown in Fig. 6.14. The drum diameter is 250 mm. The contact angle is 90°. If an operating force of 700 N is applied at the end of the lever and the coefficient of friction is 0.35 determine the torque that may be sustained by the brake. VTU February 2002

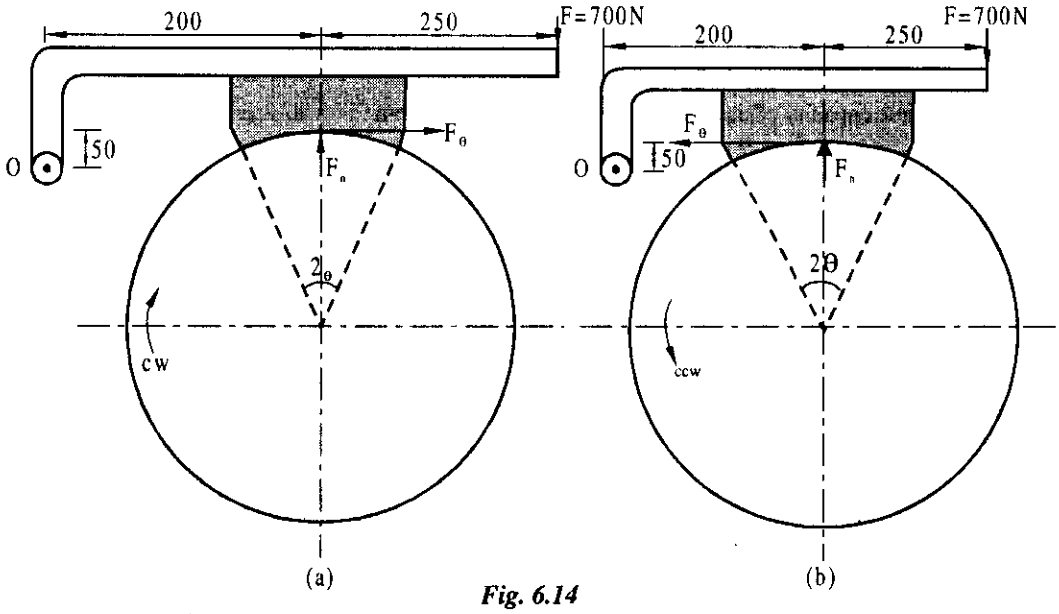


Fig. 6.14

Data :

a = 200 mm; b = 250 mm; c = 50 mm; 2θ = 90°; D = 250 mm ∴ r = 125 mm;
μ = 0.35; F = 700 N

Solution :

Since 2θ > 60°

Equivalent coefficient of friction $\mu' = \dfrac{4\mu\sin\theta}{2\theta + \sin 2\theta} = \dfrac{4\times 0.35\times\sin 45°}{90\times\dfrac{\pi}{180}+\sin 90} = 0.385$

For clockwise rotation since F_θ is away from the fulcrum (Fig. 6.14 a) from Table 19.1 (Fig. c) [i.e., use ccw formula from the table]

Applied effort or Actuating force $F = \dfrac{F_\theta . a}{a+b}\left(\dfrac{1}{\mu'} - \dfrac{c}{a}\right)$ ---- 19.148 (DDHB)

i.e., $700 = \dfrac{F_\theta \times 200}{200+250}\left[\dfrac{1}{0.385} - \dfrac{50}{200}\right]$

∴ Tangential force F_θ = 670.954 N

∴ Torque $M_t = F_\theta . r = 670.954 \times 125 = 83869.3$ Nmm

For ccw rotation since F_θ is towards the fulcrum (Fig. 6.14 b) from Table 19.1 (Fig. c) [use cw formula from the table]

$$\text{Applied effort } F = \frac{F_\theta a}{a+b}\left(\frac{1}{\mu}+\frac{c}{a}\right) \qquad \text{---- 19.147}$$

$$\text{i.e., } 700 = \frac{F_\theta \times 200}{200+250}\left(\frac{1}{0.385}+\frac{50}{200}\right)$$

$\therefore$ Tangential force F_θ = 553.136 N

$\therefore$ Torque $M_t = F_\theta . r$ = $553.136 \times 125 = 69142$ Nmm

Hence torque capacity of the given brake $M_t = 83869.3$ Nmm

[Select the bigger value as the capacity]

Example 6.19

The block brake shown in Fig. 6.15 is to balance a torque of 500 Nm on a drum shaft at 1000 rpm. Assuming the coefficient of friction between the brake shoe and drums to be 0.25, determine

(i) Tangential force on the shoe

(ii) Normal force on the shoe

(iii) Force F applied to the brake for clockwise and counter clockwise rotation.

(iv) The dimension c required to make the brake self locking assuming the other dimensions remain the same.

(v) Heat generated.

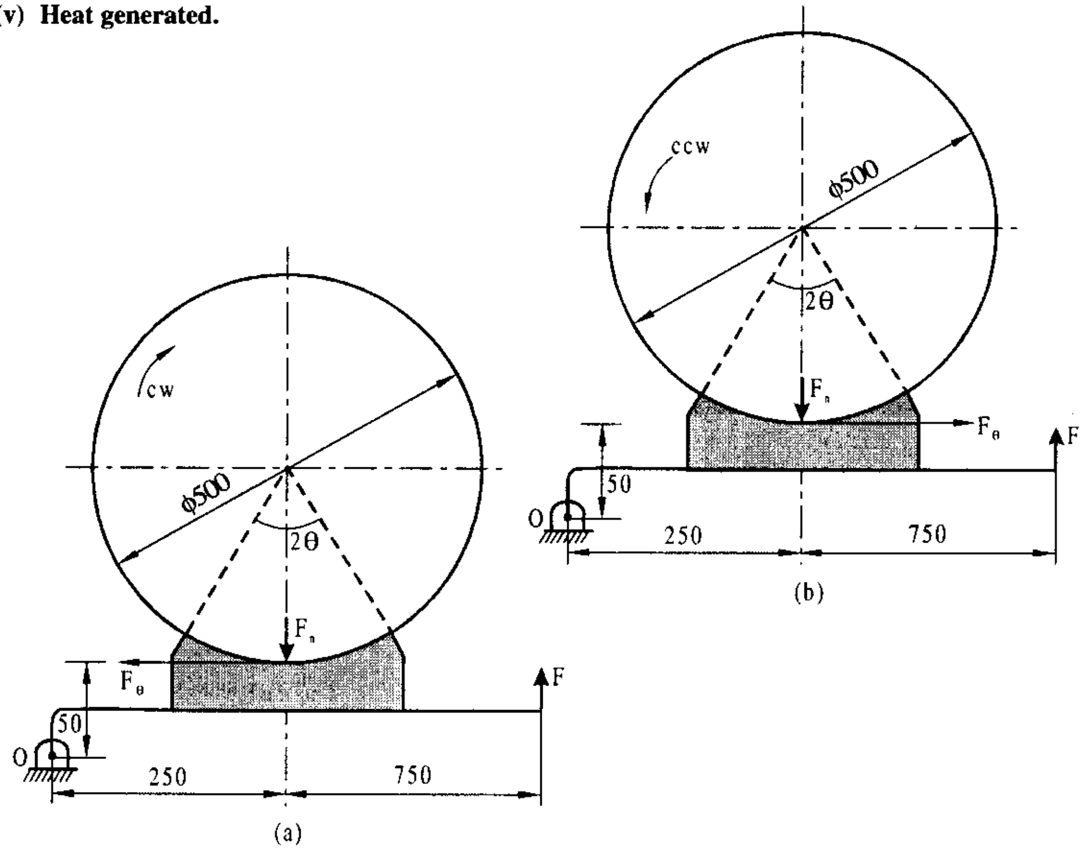


Fig. 6.15

Data :

a = 250 mm; b = 750 mm; c = 50 mm; D = 500 mm ∴ r = 250 mm;
M_t = 500 Nm = 5 × 10⁵ Nmm; n = 1000 rpm; μ = 0.25

Solution :

i) Tangential force on the shoe (F_θ)

$$\text{Torque } M_t = F_\theta . r$$

$$\text{i.e., } 5 \times 10^5 = F_\theta \times 250$$

$$\therefore F_\theta = 2000 \text{ N}$$

ii) Normal force on the shoe (F_n)

$$\text{Normal force } F_n = \frac{F_\theta}{\mu} = \frac{2000}{0.25} = 8000 \text{ N}$$

iii) Applied effort F for cw and ccw rotation

For clockwise rotation F_θ acts towards the fulcrum (Fig. 6.15 a) ∴ From Table 19.1 (Fig. b) [use c.w formula)

$$\text{Actuating force } F = \frac{F_\theta . a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right] \quad \text{---- 19.145 (DDHB)}$$

$$= \frac{2000 \times 250}{250+750}\left[\frac{1}{0.25} - \frac{50}{250}\right] = 1900 \text{ N}$$

For counter clockwise rotation F_θ acts away from the fulcrum (Figure 6.15 b) ∴ From Table 19.1 (Fig. b) [use ccw formula]

$$\text{Actuating force } F = \frac{F_\theta . a}{a+b}\left[\frac{1}{\mu} + \frac{c}{a}\right] \quad \text{---- 19.146 (DDHB)}$$

$$= \frac{2000 \times 250}{250+750}\left[\frac{1}{0.25} + \frac{50}{250}\right] = 2100 \text{ N}$$

iv) The dimension c required to make the brake self locking assuming the other dimensions remain the same.

Self locking may occur when the brake rotates in cw direction

$$\text{For self locking } F \leq O$$

$$\therefore \frac{F_\theta . a}{a+b}\left(\frac{1}{\mu} - \frac{c}{a}\right) \leq O$$

$$\text{i.e., } \frac{1}{\mu} \leq \frac{c}{a}$$

$$\text{i.e., } \frac{c}{a} \geq \frac{1}{\mu}$$

$$\text{i.e., } c \geq \frac{a}{\mu}$$

$$\geq \frac{250}{0.25}$$

$$\geq 1000$$

$\therefore$ If $c \geq 1000$ mm then self locking will occur

v) Head generated

$$\text{Heat generated } H_g = \mu p A_c v \qquad \text{---- 19.191 a (DDHB)}$$

$$= \mu F_n v \quad [\because \; p.A_c = F_n]$$

$$= 0.25 \times 8000 \times \frac{\pi \times 500 \times 1000}{60000} \quad \left(\because \; v = \frac{\pi d n}{60000}\right)$$

$$= 52359.88 \text{ watts} = 52.36 \text{ kJ/sec}$$

Example 6.20

Fig. 6.16 a shows a single block brake. The brake drum diameter is 400 mm and rotates at a speed of 150 rpm. The friction material permits a maximum pressure of 0.5 MPa and $\mu = 0.25$. Face width of the block is 50 mm. If the brake is applied for 10 secs at full capacity to bring the shaft to stop determine (i) Effort (ii) Maximum torque (iii) Heat generated.

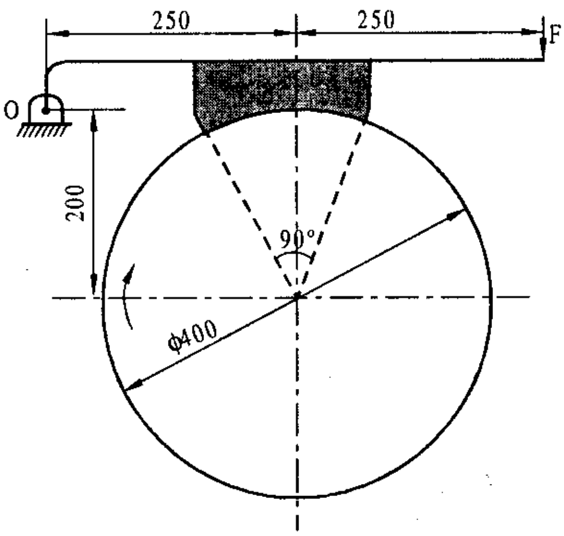


Fig. 6.16 a

Data :

$a = 250$ mm; $b = 250$ mm; $c = 0$; $n = 150$ rpm; $D = 400$ mm; $\therefore$ $r = 200$ mm; $p = 0.5$ N/mm^2

$\mu = 0.25$; $b' = 50$ mm; $t = 10$ secs; $2\theta = 90°$

Solution :

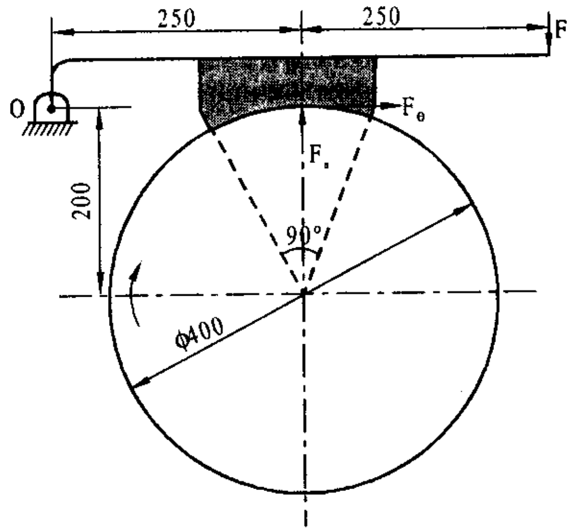


Fig. 6.16 b

For the given direction of rotation (cw) the various forces acting on the shoe are as shown in Fig. 6.16 b

i) Effort (F)

Since $2\theta > 60°$

Equivalent coefficient of friction $\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \dfrac{4 \times 0.25 \times \sin 45}{90 \times \frac{\pi}{180} + \sin 90} = 0.275$

Normal force $F_n = p\,(2\,b'\,r \sin\theta) = 0.5\,(2 \times 50 \times 200 \times \sin 45) = 7071.07$ N

Tangential force $F_\theta = \mu' F_n = 0.275 \times 7071.07 = 1944.54$ N

Since F_θ passes through the fulcrum from Table 19.1

Effort $F = F_\theta \dfrac{a}{\mu'(a+b)} = \dfrac{1944.54 \times 250}{0.275(250+250)} = 3535.534$ N ---- 19.144 (DDHB)

ii) Maximum torque

Torque $M_t = F_\theta . r = 1944.54 \times 200 = 388908$ Nmm = 388.908 Nm

iii) Heat generated

Heat generated $H_g = \mu\, p\, A_c . v$ ---- 19.191 a (DDHB)

$= \mu' F_n v \quad (\because \; p A_c = F_n)$

$= 0.275 \times 7071.07 \times \left(\dfrac{\pi \times 400 \times 150}{60000}\right) \quad \left[\because \; v = \dfrac{\pi d n}{60000}\right]$

$= 6108.96$ J/sec

$\therefore$ Heat generated during 10 secs $= 6108.96 \times 10 = 61089.6$ J $= 61.1$ kJ

Example 6.21

A single block brake with a torque capacity of 15 Nm is shown in Fig. 6.17 a. The coefficient of friction is 0.3 and the maximum pressure on the brake lining is 1 N/mm². The width of block is equal to its length calculate, (i) Actuating force (ii) Dimensions of the block (iii) Resultant hinge-pin reaction and (iv) Rate of heat generated, if the brake drum rotates at 50 rpm.

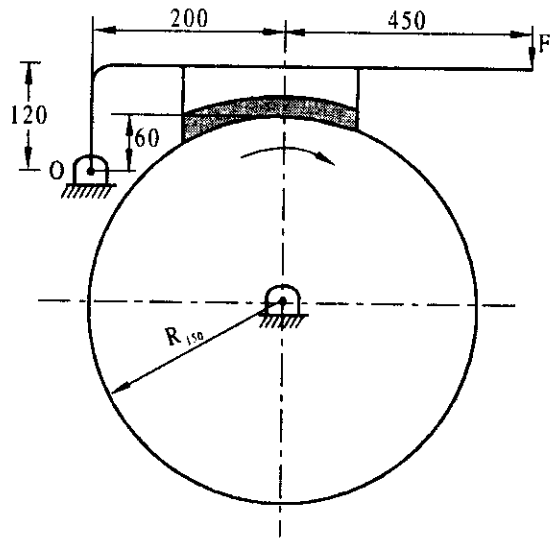


Fig. 6.17 a

Data :

$r = 150$ mm; $M_t = 15$ Nm $= 15 \times 10^3$ Nmm; $\mu = 0.3$; $p = 1$ N/mm²; $b' = l$; $n = 50$ rpm

$a = 200$ mm; $b = 450$ mm; $c = 60$ mm

Solution :

For the given direction of rotation (cw) the various forces acting on the shoe are as shown in Fig. 6.17 b.

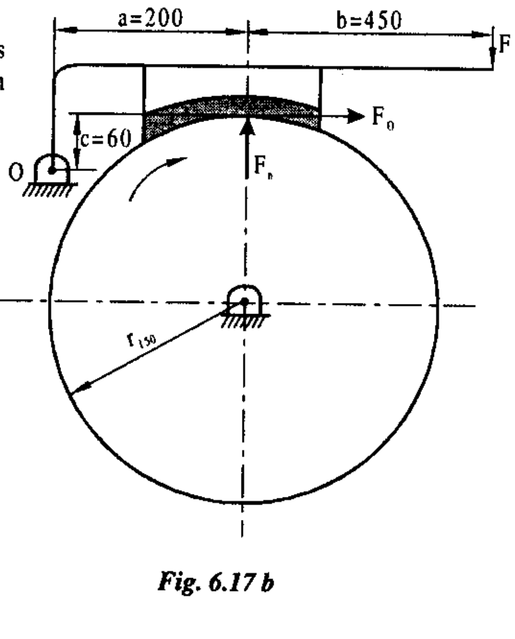


Fig. 6.17 b

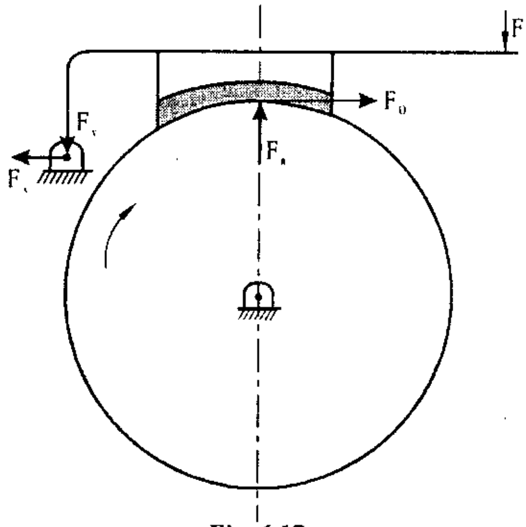


Fig. 6.17 c

i) Actuating force

Torque $M_t = F_\theta . r$

i.e., $15 \times 10^3 = F_\theta \times 150$

$\therefore F_\theta = 100$ N = Tangential force

Since F_θ is above the centre of drum and fulcrum and its direction away from the fulcrum from Table 19.1 (Fig. c) [use ccw formula from the Table)

$$\text{Actuating force } F = \frac{F_\theta . a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right] \text{ Bell crank lever} \quad \text{---- 19.148 (DDHB)}$$

$$= \frac{100 \times 200}{200+450}\left[\frac{1}{0.3} - \frac{60}{200}\right] = 93.33 \text{ N}$$

ii) Dimensions of the block

Normal force F_n = p × projected area of the block = p l b'

Spring

i.e., $\frac{F_\theta}{\mu} = p \times l \times l$ [$\because$ $F_\theta = \mu F_n$ and $b' = l$]

i.e., $\frac{100}{0.3} = 1 \times l^2 \therefore l = 18.2574 \text{ mm} \approx 18.26 \text{ mm}$

$\therefore$ Width of shoe b' = 18.26 mm

Block or shoe

Length of shoe l = 18.26 mm

iii) Resultant hinge pin reactions (Refer Fig. 6.17 c)

$$\text{Normal force } F_n = \frac{F_\theta}{\mu} = \frac{100}{0.3} = 333.33 \text{ N}$$

For the equilibrium of lever with block, sum of the vertical and horizontal forces must be equal to zero.

Considering the vertical forces

$$F + F_y - F_n = 0$$

i.e., $93.33 + F_y - 333.33 = 0$

$$\therefore \; F_y = 240 \text{ N}$$

Considering the horizontal forces

$$F_\theta - F_x = 0$$

i.e., $100 - F_x = 0 \;\; \therefore \;\; F_x = 100 \text{ N}$

$\therefore$ Resultant hinge pin reaction $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{100^2 + 240^2} = 260 \text{ N}$

iv) Rate of heat generated

Heat generated $H_g = \mu \, p \, A_c v = \mu F_n v \quad (\because \; pA_c = F_n)$ ---- 19.191 a (DDHB)

$$= 0.3 \times 333.33 \times \left(\frac{\pi \times 300 \times 50}{60000}\right) = 78.54 \text{ J/sec}$$

6.18 DOUBLE SHOE OR BLOCK BRAKES

Although the single block brake is simple and reliable, the major problem is the unbalanced normal force F_n which exerts a heavy pressure on the shaft bearings and causes the bending of the shaft. In order to overcome this drawback, a double block or shoe brake is used as shown in Fig. 6.18. It consists of two shoes placed at the opposite ends of the diameter of the wheel. The brake is set by a spring which pulls the upper ends of the brake arms together. It is normally used in electric cranes. The braking action of the double shoe brake is doubled by the use of two blocks. Therefore the braking torque $M_t = (F_{\theta 1} + F_{\theta 2})\, r$.

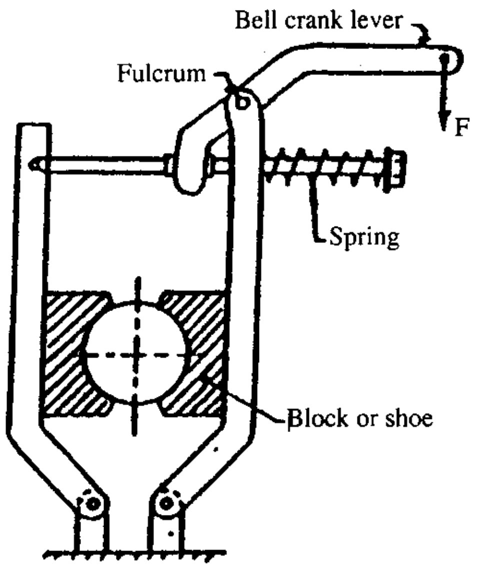


Fig. 6.18 : Double block or shoe brake

6.19 PROCEDURAL STEPS FOR THE DESIGN OF DOUBLE SHOE BRAKE

The various forces acting on the shoes for the clockwise rotation of the brake drum is shown in Fig. 6.19

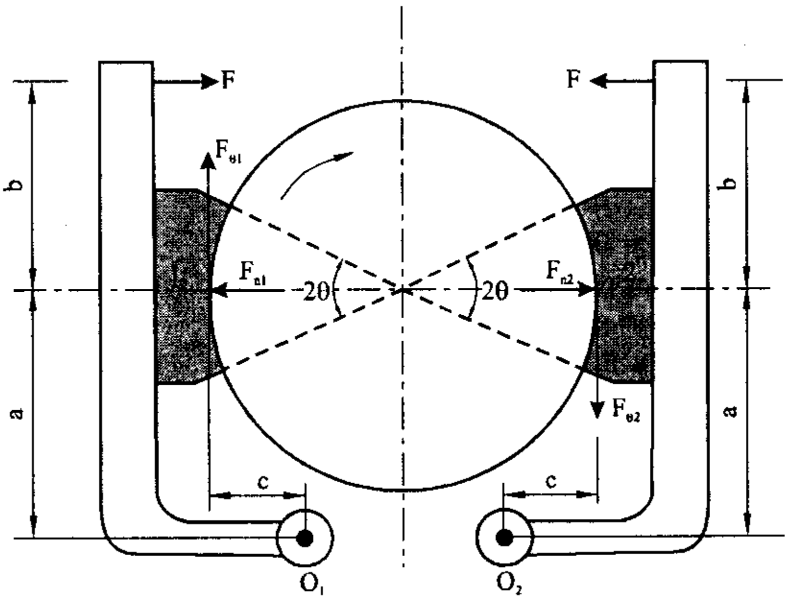


Fig. 6.19

Let F = Spring Force

$F_{\theta 1}$ = Tangential force on the left hand side shoe

$F_{\theta 2}$ = Tangential force on the right hand side shoe

F_{n1} = Normal force on the left hand side shoe

F_{n2} = Normal force on the right hand side shoe

l = Length of shoe

b' = Width of shoe

b_1 = Width of lever

h_1 = Thickness or Depth of lever

i) Torque transmitted

$$\text{Torque } M_t = 9550 \times 1000 \times \frac{N}{n} \text{ where } M_t \text{ in Nmm}$$

ii) Actuating or spring force and tangential force on the shoes

(a) Consider the left hand shoe

$F_{\theta 1}$, lies above the centre of drum and fulcrums and acting away from the fulcrum

∴ From Table 19.1 [Fig.c, ccw formula)

$$\text{Actuating force } F = \frac{F_{\theta 1} a}{a+b}\left(\frac{1}{\mu} - \frac{c}{a}\right) \quad \text{---- 19.148}$$

∴ Find $F_{\theta 1}$ in terms of F ---- (i)

b) Consider the right hand shoe

$F_{\theta 2}$ lies above the centre of drum and fulcrum and acting towards the fulcrum

∴ From Table 19.1 [Fig. c, cw formula]

$$\text{Actuating force } F = \frac{F_{\theta 2} a}{a+b}\left[\frac{1}{\mu}+\frac{c}{a}\right] \quad \text{---- 19.147}$$

Find $F_{\theta 2}$ in terms of F ---- (ii)

$$\text{Now } M_t = (F_{\theta 1} + F_{\theta 2})\, r$$

Substituting the values of $F_{\theta 1}$ and $F_{\theta 2}$, we can find F. Now substituting the values of F in equations (i) and (ii), we can get $F_{\theta 1}$ and $F_{\theta 2}$.

If $2\theta > 60°$, then use μ' instead of μ

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

iii) Normal Force on the shoes

$$F_{n1} = \frac{F_{\theta 1}}{\mu} \text{ and } F_{n2} = \frac{F_{\theta 2}}{\mu}$$

If $2\theta > 60°$ then instead of μ, μ' is used.

iv) Design of shoe

Projected bearing area for one shoe $A_b = b'l$ where projected length of shoe $l = 2r \sin\theta$

and width of shoe $b' = \frac{F_{n\,max}}{l.p}$, p = normal or bearing pressure on the shoes and $F_{n\,max}$ is the larger among the two values of F_{n1} and F_{n2}.

v) Design of lever

Assuming maximum bending moment at the pivot, neglecting the effect of F_n and F_θ,

Maximum bending moment $M_b = F\,(a + b)$

We have $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_l h_l^3}{12}$, $c = \frac{h_l}{2}$ and σ_b = Permissible bending stress on the lever material.

∴ Width of lever b_l and thickness of lever h_l

vi) Heat generated

Heat generated on left shoe per unit time $H_g = \mu p A_c v = \mu F_{n1} v$ where $v = \frac{\pi d n}{60{,}000}$ ---- 19.191a

Heat generated on right shoe per unit time $H_g = \mu F_{n2} v$

If $2\theta > 60°$ then instead of μ, μ' is used.

Example 6.22

Fig 6.20a shows a double shoe brake on a drum diameter of 320mm. The angle of contact of each shoe = 110°. The brake has to absorb 10kW at 1000 rpm. Design the brake and determine the spring force. The brake lever is rectangular cross section whose height is 3 times its width $\mu = 0.42$

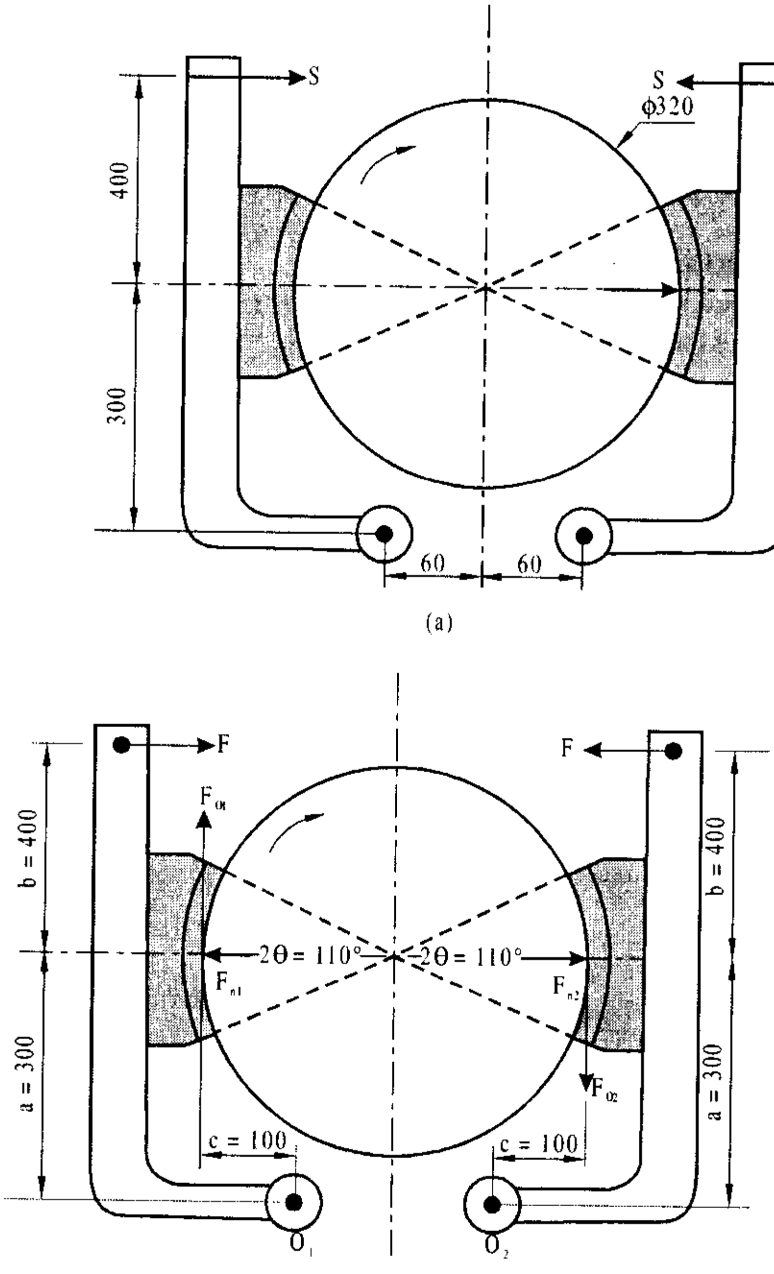


Fig. 6.20

Data :

D =320 mm; ∴ r = 160 mm; 2θ = 110°; N = 10 kW; n = 1000 rpm; μ = 0.42;
a =300 mm; b = 400 mm; c = 160 – 60 = 100 mm; h_1 = 3b.

Solution :

i. Torque transmitted

$$M_t = 9550\times1000\times\frac{N}{n} = 9550\times1000\times\frac{10}{1000} = 95500\,\text{Nmm}$$

ii. Actuating force or spring force and tangential force on the shoes

The various forces acting on the shoes for the given direction of rotation (cw) of the drum are shown in Fig. 6.20b.

a) Consider the left hand shoe

$F_{\theta 1}$ lies above the centre of drum and fulcrum and acting away from the fulcrum

∴ From Table 19.1 (Fig. c, ccw formula)

$$\text{Actuating force } F = \frac{F_{\theta 1}a}{a+b}\left(\frac{1}{\mu'}-\frac{c}{a}\right) \quad \text{---- 19.148}$$

Since 2θ > 60°

$$\mu' = \frac{4\mu\sin\theta}{2\theta+\sin 2\theta} = \frac{4\times0.42\times\sin 55}{110\times\frac{\pi}{180}+\sin 110} = 0.48126$$

$$\therefore F = \frac{F_{\theta 1}\times300}{300+400}\left[\frac{1}{0.48126}-\frac{100}{300}\right]$$

i.e.,

$$F_{\theta 1} = 1.3375\,F \quad \text{----(i)}$$

b) Consider the right hand shoe

$F_{\theta 1}$ lies above the centre of drum and fulcrum and acting towards the fulcrum. ∴ From Table 19.1 [Fig.c, cw formula]

$$\therefore \text{Actuating force } F = \frac{F_{\theta 2}.a}{a+b}\left(\frac{1}{\mu'}+\frac{c}{a}\right) \quad \text{---- 19.147}$$

$$= \frac{F_{\theta 2}\times300}{300+400}\left[\frac{1}{0.48126}+\frac{100}{300}\right]$$

$$\text{i.e. } F_{\theta 2} = 0.9677\,F \quad \text{----(ii)}$$

Now

$$M_t = (F_{\theta 1}+F_{\theta 2})\,r$$

$$\text{i.e., } 95500 = (1.3375\,F+0.9677\,F)\,160$$

$$\therefore \text{Actuating force } F = 258.926\,\text{N} = \text{Spring force}$$

Hence, tangential force on left shoe $F_{\theta 1} = 1.3375\times258.926 = 346.3$ N

Tangential force on right shoe $F_{\theta 2} = 0.9677\times258.926 = 250.6$ N

iii. Normal force on the shoes

$$\text{Normal force on the left shoe } F_{n1} = \frac{F_{\theta 1}}{\mu'} = \frac{346.3}{0.48126} = 720.2\,\text{N}$$

Normal force on the right shoe $F_{n2} = \frac{F_{02}}{\mu'} = \frac{250.6}{0.48126} = 520.72$ N

iv. Design of shoe

Maximum normal force $F_{max} = 720.2$ N

Projected length of shoe $l = 2r\sin\theta = 2 \times 160 \times \sin 55 = 262$ mm

Width of shoe $b' = \frac{F_{max}}{lp}$ where p = normal pressure = 0.2 N/mm² (assume)

$$= \frac{720.2}{262 \times 0.2} = 13.744 \text{ mm} \approx 14 \text{ mm}$$

v. Design of lever

Assuming maximum bending moment at the pivot and neglecting the effect of F_n and F_0

Maximum bending moment $M_b = F(a + b) = 258.926(300 + 400) = 181248.2$ Nmm

Assume C40 steel as lever material and FOS = 2.5

∴ From Table 1.5 (Old DDHB) for C40 steel

$$\sigma_y = 328.6 \text{ MPa}$$

$$\therefore \sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{2.5} = 131.44 \text{ N/mm}^2 = \sigma_b$$

$$I = \frac{b_l h_l^3}{12} = \frac{b_l (3b_l)^3}{12} = \frac{27 b_l^4}{12} \qquad (\because h_l = 3b_l)$$

$$c = \frac{h_l}{2} = \frac{3b_l}{2} = 1.5\, b_l$$

We have $\frac{M_b}{I} = \frac{\sigma_b}{c}$

i.e., $\frac{181248.2}{\frac{27}{12} b_l^4} = \frac{131.44}{1.5 b_l}$ i.e., $b_l = 9.7234$ mm

Select width of lever b_l = 10 mm

∴ Height of lever h_l = 30 mm $(\because h_l = 3b_l)$

vi) Heat generated

Heat generated on the left shoe per unit time $= \mu' p A_c v = \mu' F_{n1} v$

$$= 0.48126 \times 720.2 \times \frac{\pi \times 320 \times 1000}{60{,}000} = 5807.4 \text{ J}$$

Heat generated on the right shoe per unit time

$$= \mu' F_{n2} v = 0.48126 \times 520.72 \times \frac{\pi \times 320 \times 1000}{60{,}000} = 4198.9 \text{ J}$$

∴ Total heat generated per unit time $H_g = 5807.4 + 4198.9 = 10006.3$ J = 10 kJ or 10 kW

Example 6.23

A double block brake drum shown in Fig. 6.21a has a drum diameter of 300 mm, contact angle of each shoe = 90° and μ = 0.4. Find the spring force necessary to absorb a torque of 30 Nm.

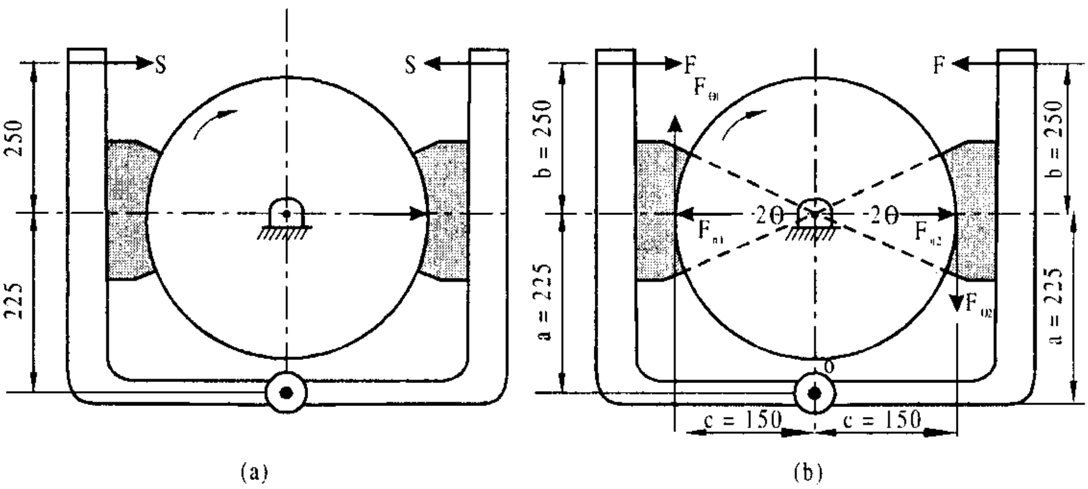


Fig. 6.21

Data :

$$\mathbf{D = 300\ mm \therefore r = 150\ mm;\ 2\theta = 90°;\ \mu = 0.4;\ M_t = 30\ Nm = 30 \times 10^3\ Nmm;}$$

$$\mathbf{a = 225\ mm;\ b = 250mm;\ c = 150\ mm}$$

Solution :

Spring force

The various forces acting on the shoes for the given direction of rotation (cw) of the drum are shown in Fig. 6.21b.

Consider the left hand shoe

F_{θ_1} lies above the centre of drum and fulcrum and acting away from the fulcrum. From Table 19.1 [Fig.c, ccw formula]

$$\text{Actuating force } F = \frac{F_{\theta 1} a}{a+b}\left(\frac{1}{\mu'} - \frac{c}{a}\right) \qquad \text{---- 19.148}$$

$$\text{Since } 2\theta > 60°,\ \mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 45}{90 \times \frac{\pi}{180} + \sin 90} = 0.44$$

$$\therefore F = \frac{F_{\theta 1} \times 225}{225 + 250}\left[\frac{1}{0.44} - \frac{150}{225}\right]$$

$$\text{i.e., } \mathbf{F_{\theta 1} = 1.3145F}$$

Consider the right shoe

F_{02} lies above the centre of drum and fulcrum and acting towards the fulcrum.

From Table 19.1 (Fig c, cw formula)

$$F = \frac{F_{02}a}{a+b}\left[\frac{1}{\mu'}+\frac{c}{a}\right] \quad \text{---- 19.147}$$

$$= \frac{F_{02}\times 225}{225+250}\left[\frac{1}{0.44}+\frac{150}{225}\right]$$

$$\therefore \mathbf{F_{02} = 0.7182\,F}$$

Now Torque transmitted $M_t = (F_{01}+F_{02})\,r$

i.e., $30\times 10^3 = (1.3145\,F + 0.7182\,F)\,150$

$\therefore$ Spring force or Actuating force $F = 98.4$ N

Example 6.24

A double block brake is shown in Fig. 6.22a. The drum rotates at 200 rpm when the applied force is 1000 N and $\mu = 0.25$. Determine (i) Braking torque (ii) Power lost as heat (iii) Amount of heat generated.

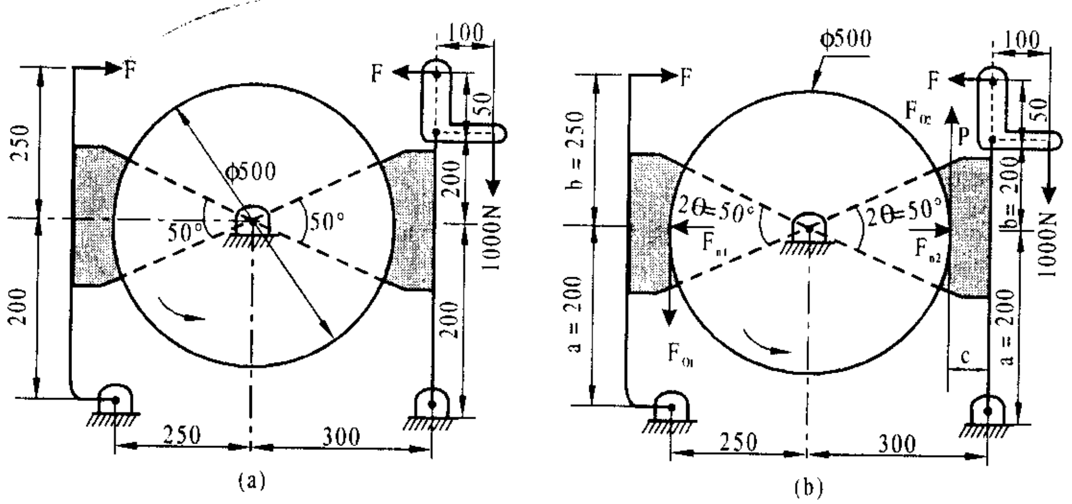


Fig. 6.22

Data :

$$\mathbf{n = 200\ rpm;\ \mu = 0.25;\ 2\theta = 50°;\ D = 500\ mm\ \therefore\ r = 250\ mm}$$

Solution :

The various forces acting on the shoes for the given direction of rotation are shown in Fig. 6.22b. Taking moments about P

$$1000\times 100 = F\times 50$$

$\therefore$ Actuating force $F = 2000$ N

i) Braking torque

Consider the left shoe

$$a = 200\text{mm};\ b = 250\text{mm};\ c = 0$$

$F_{\theta 1}$ passes through the fulcrum. From Table 19.1 (Fig. a)

$$\text{Actuating force } F = \frac{F_{\theta 1} a}{\mu(a+b)} \quad \text{---- 19.144}$$

$$\text{i.e., } 2000 = \frac{F_{\theta 1} \times 200}{0.25(200+250)} \quad \therefore F_{\theta 1} = 1125\text{ N}$$

Consider right shoe

$a = 200$ mm; $b = 200$ mm; $c = 300 - 250 = 50$ mm

$F_{\theta 2}$ lies between the fulcrum and the centre of drum and acting away from the fulcrum.

From Table 19.1 (Fig. b ccw formula)

$$\text{Actuating force } F = \frac{F_{\theta 2} a}{(a+b)}\left[\frac{1}{\mu} + \frac{c}{a}\right] \quad \text{---- 19.146}$$

$$\text{i.e., } 2000 = \frac{F_{\theta 2} \times 200}{(200+200)}\left(\frac{1}{0.25} + \frac{50}{200}\right)$$

$$\therefore F_{\theta 2} = 941.2\text{ N}$$

Now

$$\text{Braking torque } M_t = (F_{\theta 1} + F_{\theta 2})r = (1125 + 941.2)250 = 516550\text{ Nmm} = 516.55\text{ Nm}$$

ii) Power lost as heat

$$M_t = 9550 \times \frac{N}{n} \text{ where } M_t \text{ in Nm}$$

$$\text{i.e., } 516.55 = 9550 \times \frac{N}{200}$$

$$\therefore N = 10.82\text{ kW}$$

iii) Amount of heat generated

Total heat generated = Heat generated on the left shoe + Heat generated on the right shoe = $(\mu p A_c v)_L + (\mu p A_c v)_R$

$$= \mu F_{n1} v + \mu F_{n2} v = F_{\theta 1} v + F_{\theta 2} v \qquad [\because F_\theta = \mu F_n \text{ and } pA_c = F_n]$$

$$= (F_{\theta 1} + F_{\theta 2})v = (1125 + 941.2)\left(\frac{\pi \times 500 \times 200}{60{,}000}\right)$$

$$= 10818.6\text{ W or J} = 10.82\text{ kW or kJ per unit time}$$

Example 6.25

A double shoe brake shown in Fig. 6.23 is capable of absorbing a torque of 1500 Nm. The diameter of brake drum = 400 mm and angle of contact for each shoe = 100°. If the coefficient of friction between the brake drum and the lining in 0.4, find

i) Spring force necessary to set the brake

ii) width of brake shoe if the bearing pressure on the lining material is not to exceed 0.3 MPa

iii) Design the lever. Take C40 steel for lever and factor of safety = 3 **BU Aug. 1995**

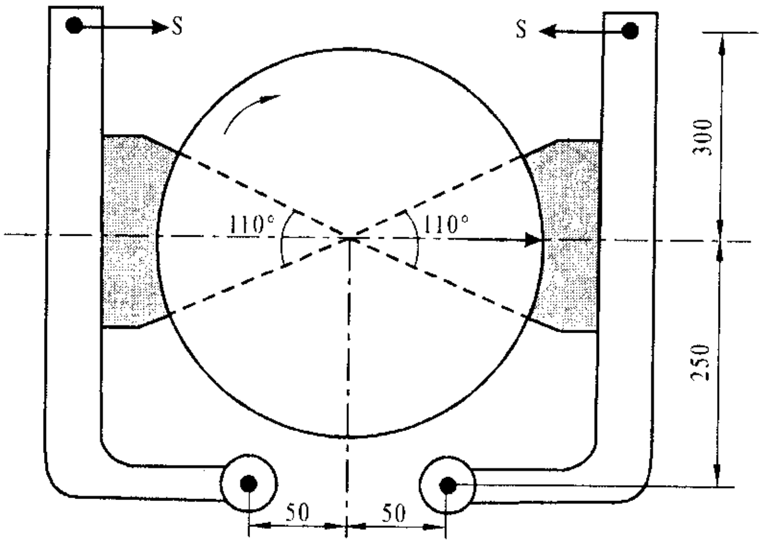


Fig. 6.23

Solution : Similar to Example 6.22

Example 6.26

The block brake shown in Fig. 6.24 below has two shoes each subtending an angle 80°; the coefficient of friction of the brake material is 0.3. The brake drum diameter is 800 mm while the rope drum measures 1200 mm in diameter. It is required to stop the load of 20 kN lowering at a velocity of 5 m/s in a distance of 20 meters. Determine; the effort E required at the end of the lever, the width of the brake shoes given the permissible pressure of 0.7 MPa and the amount of heat generated.

(VTU, July 2006)

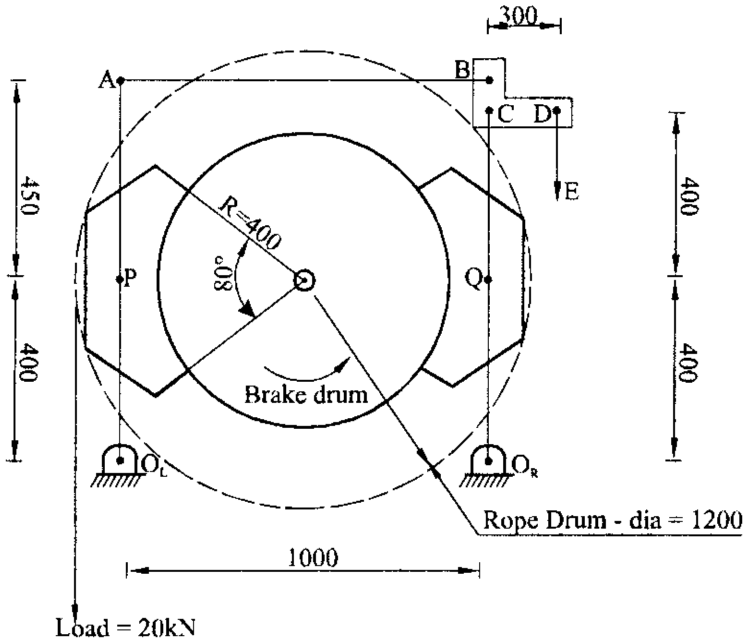


Fig. 6.24

Data:

$2\theta = 80°$, $\therefore \theta = 40°$; $\mu = 0.3$; D = 800 mm, $\therefore$ r = 400mm;

Diameter of rope drum = 1200mm

Load on the rope drum = 20kN = 20,000 N

Velocity v = 5m/sec; Distance moved by the load = 20m

Permissible pressure p = 0.7 MPa = 0.7 N/mm^2.

Solution :

(a) Effort E required at the end of the lever

(i) Torque Transmitted

Torque on the rope drum shaft = Load on the rope drum x Radius of rope drum

$$= 20{,}000 \times \frac{1200}{2} = 12 \times 10^6 \text{ N.mm}$$

Since the rope drum and the brake drum are on the same shaft, braking torque $M_t = 12 \times 10^6$ N.mm

(ii) Consider the left shoe

The various forces acting on the shoes for the given direction of rotation are shown in Fig. 6.25.

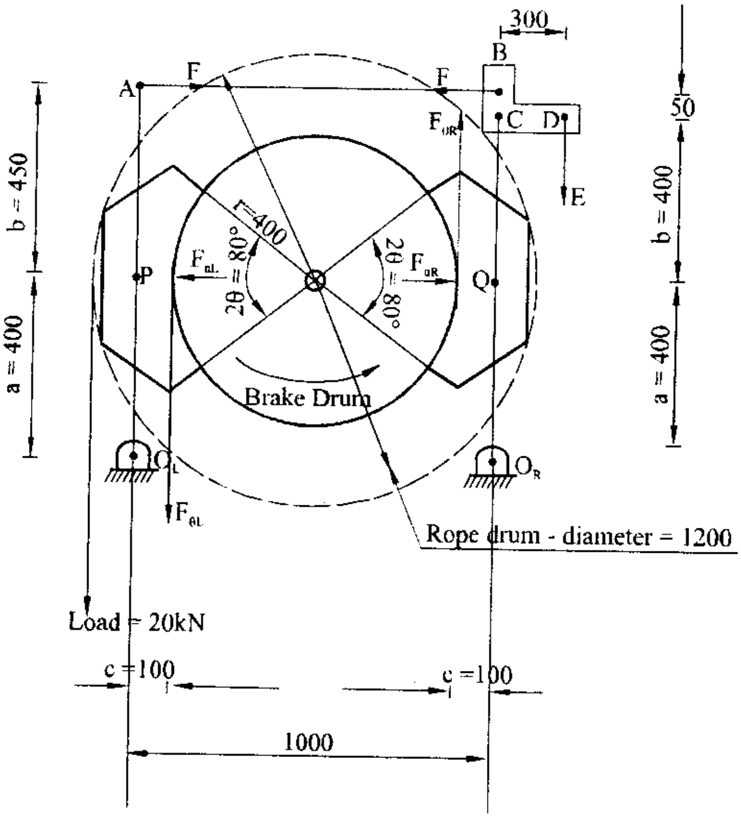


Fig. 6.25

Since 20 > 60°

∴ Equivalent coefficient of friction $\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$ where μ = Actual coefficient of friction

$$= \frac{4 \times 0.3 \times \sin 40°}{80 \times \dfrac{\pi}{180} + \sin 80} = 0.324$$

F_{θ_L} lies between the centre of drum and the fulcrum and acting towards the fulcrum.

Taking moments about O_L

$$F \times (a + b) + F_{\theta_L} \times c = F_{n_L} \times a$$

$$= \frac{F_{\theta_L}}{\mu'} \times a \quad (\because F_{\theta_L} = \mu' F_{n_L})$$

$$\text{i.e., } F(a + b) = \frac{F_{\theta_L}}{\mu'} . a - F_{\theta_L} . c$$

$$\therefore \text{ Actuating force } F = \frac{F_{\theta_L}.a}{(a+b)}\left[\frac{1}{\mu'}-\frac{c}{a}\right] \quad \text{---- 19.145}$$

$$\text{i.e., } F = \frac{F_{\theta_L}\times 400}{(400+450)}\left[\frac{1}{0.324}-\frac{100}{400}\right] = 1.3348\, F_{\theta_L}$$

$$\therefore F_{\theta_L} = 0.7492F$$

(iii) Consider the right shoe.

F_{θ_R} Lies between the centre of drum and the fulcrum, and acting away from the fulcrum.

Considering equilibrium of horizontal forces, Force acting on pin C = Force acting on pin B = F Taking moments about O_R

$$F(a+b) = F_{\theta_R} \times c + F_{n_R} \times a$$

$$= F_{\theta_R}.c + \frac{F_{\theta_R}}{\mu'}.a \, (\because F_{\theta_R} = \mu' F_{n_R})$$

$$\therefore \text{ Actuating force } F = \frac{F_{\theta_R}.a}{(a+b)}\left[\frac{1}{\mu'}+\frac{c}{a}\right] \quad \text{---- 19.146}$$

$$\text{i.e., } F = \frac{F_{\theta_R}\times 400}{(400+400)}\left[\frac{1}{0.324}+\frac{100}{400}\right] = 1.66821\, F_{\theta_R}$$

$$\therefore F_{\theta_R} = 0.59944F$$

$$\text{Braking torque } M_t = (F_{\theta_L} + F_{\theta_R})\, r$$

$$\text{i.e., } 12\times 10^6 = (0.7492F + 0.59944F)400$$

$$\therefore \text{ Actuating force } F = 22244.6\text{N}$$

Taking moments about C

$$F\times 50 = E\times 300$$

$$\text{i.e., } 22244.6\times 50 = E\times 300$$

$\therefore$ Effort E required at the end of the lever = 3707.4N

(b) Width of brake shoe

Tangential force on the left shoe $F_{\theta_L} = 0.7492\,F = 0.7492 \times 22244.6 = 16665.65$ N

Tangential force on the right shoe $F_{\theta_R} = 0.59944\,F = 0.59944 \times 22244.6 = 13334.3$

Normal force on the left shoe $F_{n_L} = \frac{F_{\theta_L}}{\mu'} = \frac{16665.65}{0.324} = 51437.2$ N

Normal force on the right shoe $F_{n_R} = \frac{F_{\theta_R}}{\mu'} = \frac{13334.3}{0.324} = 41155.25$ N

$\therefore$ Maximum normal force F_{max} = 51437.2 N

Projected length of shoe l = $2r \sin\theta = 2 \times 400 \times \sin 40 = 514.23$ mm

$\therefore$ Width of shoe b' = $\frac{F_{max}}{lp} = \frac{51437.2}{514.23 \times 0.7} = 142.9$ mm $\cong 143$ mm

(c) Heat generated

Total heat generated per unit time = Heat generated on the left shoe + Heat generated on the right shoe

$= (\mu' pA_c v)_L + (\mu' pA_c v)_R = \mu' F_{n_L} v + \mu' F_{n_R} v$

$= F_{\theta_L} v + F_{\theta_R} v$ ($\because F_\theta = \mu' F_n$ and $p A_c = F_n$)

$= (F_{\theta_L} + F_{n_R}) v = (16665.65 + 13334.3)\, 5$

= 149999.75 Watts/sec or Joules / sec

$\cong$ 150000 Watts / sec or Joules / sec

$\cong$ 150 kW/ sec or kJ / sec

Time taken by the drum to come to rest $t = \frac{20}{v} = \frac{20}{5} = 4$ secs

$\therefore$ Total amount of heat generated = 150 × 4 = 600 kW or kJ.

6.20 BAND BRAKE

A band brake consists of a flexible band generally made of steel strip lined with friction material is wrapped partly round the drum. The band is fixed and the drum rotates. The frictional force between the drum and the band will introduce brake torque on the drum. It is classified into two types (i) simple band brake (ii) Differential band brake.

6.21 SIMPLE BAND BRAKE

A simple band brake is shown in Fig. 6.26. If one end of the band is connected to the fixed fulcrum and the other end on the lever, then the band brake is called simple band brake. There is no self locking in simple band brake.

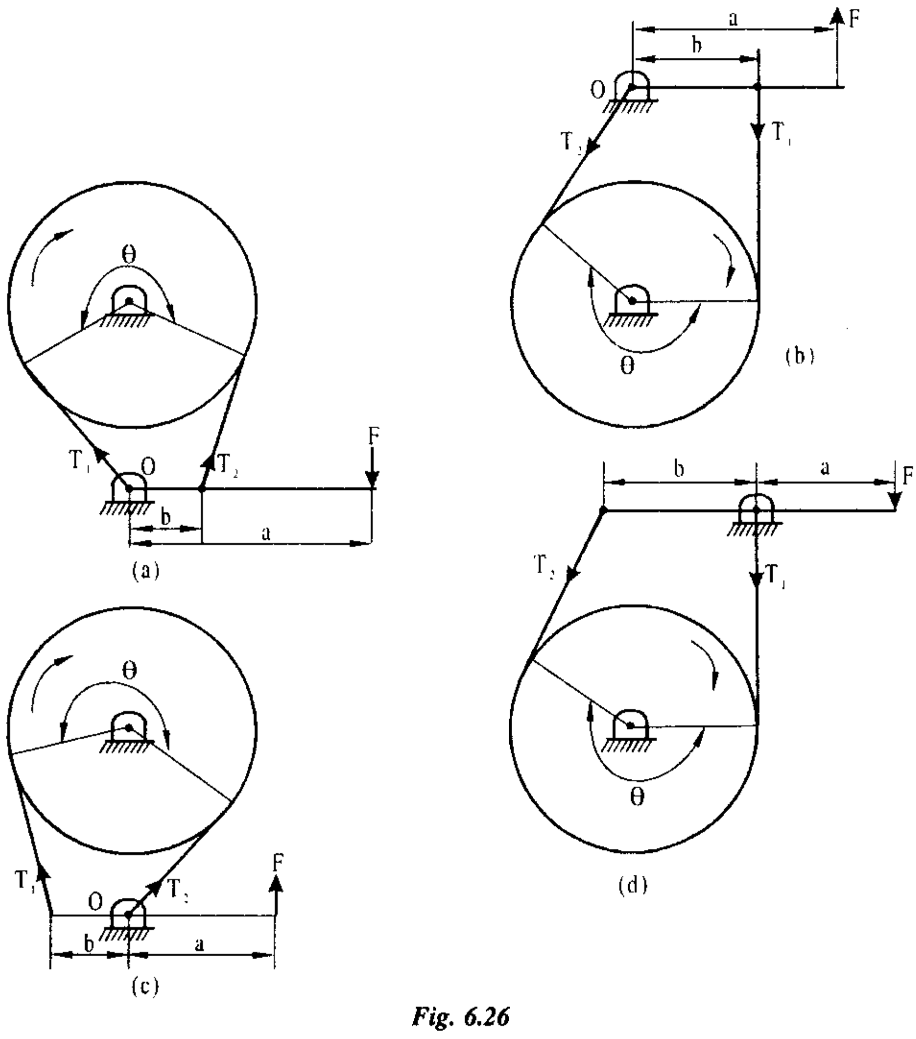


Fig. 6.26

6.22 PROCEDURAL STEPS FOR THE DESIGN OF SIMPLE BAND BRAKE

Consider a simple band brake as shown in Fig. 6.26a. The working of steel band is similar to that of a flat belt.

Let T_1 = Tension on tight side in N

T_2 = Tension on slack side in N

θ = Angle of lap in radians

μ = Coefficient of friction

M_t = Torque transmitted in Nmm

N = Power in kW

n = Speed in rpm

D = Diameter of brake drum
R = Radius of brake drum
F = Applied force in N
F_θ = Tangential force in N
h = Thickness of band
w = Width of band
σ_d = Allowable stress in the band in N/mm^2
b_l = Width of lever
h_l = Thickness of lever
σ_b = Allowable stress in the lever material in N/mm^2

i) Torque transmitted

$$M_t = 9550 \times 1000 \times N/n \text{ where } M_t \text{ in Nmm} \qquad \text{---- 19.3c}$$

ii) Diameter of the shaft

$$d = \sqrt{\frac{16M_t}{\pi\tau_s\eta}} \qquad \text{---- 19.49}$$

where τ_s = Allowable stress in the shaft material and η = keyway factor

iii) Operating force (F)

Case 1 : Clockwise rotation of the drum [Refer Fig. 6.26a]

Method - I

$$\text{Torque } M_t = (T_1 - T_2)\,R$$

Also $\frac{T_1}{T_2} = e^{u\theta}$ where θ is angle of lap in radians. Using the above two relations T_1 and T_2 can be calculated.

Now taking moments about O

$F \times a = T_2 \times b \;\; \therefore$ Operating force $F = T_2\left(\frac{b}{a}\right)$

Method - II

Torque $M_t = F_\theta \cdot R$

Using the above relation F_θ can be calculated.

$$\text{Also } F_\theta = T_1 - T_2 = T_2\left[\frac{T_1}{T_2} - 1\right] = T_2\left[e^{u\theta} - 1\right]$$

$$\therefore T_2 = \frac{F_\theta}{e^{u\theta} - 1}$$

Taking moments about O

$$F \times a = T_2 \times b$$

$\therefore$ Operating force $F = T_2 \dfrac{b}{a} = \dfrac{F_\theta}{e^{u\theta} - 1} \dfrac{b}{a}$

i.e., $$F = \frac{F_\theta . b}{a}\left[\frac{1}{e^{u\theta} - 1}\right] \quad \text{---- 19.151 (DDHB)}$$

Case 2 : Counter clockwise rotation of drum (Refer Fig. 6.27)

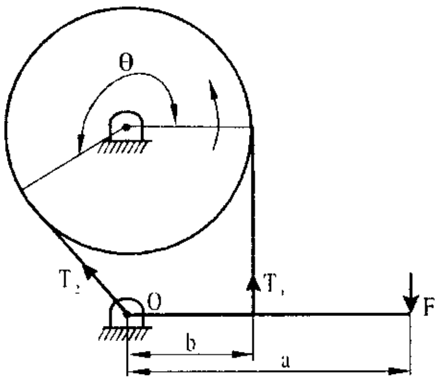


Fig. 6.27

Method I

Taking moments about O

$$F \times a = T_1 \times b \quad \therefore \text{ operating force } F = T_1\left(\frac{b}{a}\right)$$

Method II

$$F_\theta = T_1 - T_2 = T_1\left(1 - \frac{T_2}{T_1}\right) = T_1\left(1 - \frac{1}{e^{u\theta}}\right) = T_1\left(\frac{e^{u\theta} - 1}{e^{u\theta}}\right)$$

$$\therefore T_1 = F_\theta\left(\frac{e^{u\theta}}{e^{u\theta} - 1}\right)$$

Taking moments about O

$$F \times a = T_1 \times b$$

$$\therefore F = T_1 \frac{b}{a} = F_\theta\left(\frac{e^{u\theta}}{e^{u\theta} - 1}\right) \cdot \frac{b}{a}$$

$\therefore$ Operating force $$F = \frac{F_\theta . b}{a}\left(\frac{e^{u\theta}}{e^{u\theta} - 1}\right) \quad \text{---- 19.152 (DDHB)}$$

iv) Design of band

Thickness of band $h = 0.005D$ ---- 19.160 (DDHB)

Width of band $w = \frac{T_1}{h\sigma_d}$ ---- 19.161 (DDHB)

v) Design of lever

Considering the lever as a cantilever and neglecting the effect of T_1 or T_2, maximum bending moment $M_b = F_{max}$ a where F_{max} is the larger among the two values of F (cw) and F (ccw)

We have $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_1 h_1^3}{12}$ and $c = \frac{h_1}{2}$

∴ Width of lever b_1 and thickness of lever h_1

Example 6.27

A simple band brake of drum diameter 600 mm has a band passing over it with an angle of contact of 225°, while one end is connected to the fulcrum, the other end is connected to the brake lever at a distance of 400mm from the fulcrum. The brake lever is 1m long. The brake is to absorb a power of 15 kW at 720 rpm. Design the brake lever of rectangular cross-section, assuming depth to be thrice the width. Take allowable stress 80 MPa. **VTU July/Aug. 2003**

Data:

$D = 600$ mm ∴ $R = 300$ mm; $\theta = 225°$; $N = 15$ kW; $n = 720$ rpm;
$h_1 = 3b_1$; $\sigma_b = 80$ MPa

Solution:

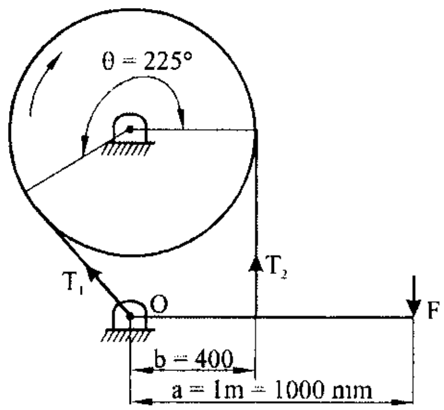


Fig. 6.28

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{15}{720} = 198958.33$ Nmm

Also $M_t = F_\theta.R$

i.e., $198958.33 = F_\theta \times 300$ ∴ Tangential force $F_\theta = 663.2$ N

$$e^{\mu\theta} = e^{0.3 \times 225 \times \pi/180} = 3.2482 \text{ Assume } \mu = 0.3$$

For cw rotation, actuating force $F = \frac{F_\theta . b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$ ---- 19.151 (DDHB)

$$= \frac{663.2 \times 400}{1000}\left[\frac{1}{3.2482-1}\right] = 118\,N$$

For ccw rotation, actuating force $F = \frac{F_\theta . b}{a}\left[\frac{e^{u\theta}}{e^{u\theta}-1}\right]$ ---- 19.152 (DDHB)

$$= \frac{663.2 \times 400}{1000}\left[\frac{3.2482}{3.2482-1}\right] = 383.3\ N$$

$\therefore\ F_{max} = 383.3\,N$

Considering the lever as a cantilever and neglecting the effect of T_1 or T_2, maximum bending moment $M_b = F_{max}.a = 383.3 \times 1000 = 383300$ Nmm

We have $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_l h_l^3}{12} = \frac{b_l (3b_l)^3}{12} = \frac{27 b_l^4}{12}$

$c_l = \frac{h_l}{2} = \frac{3b_l}{2} = 1.5\,b_l$

$\therefore\ \frac{383300}{\frac{27}{12}b_l^4} = \frac{80}{1.5 b_l}$ $\therefore\ b_l = 14.727$ mm

Select, Width of lever b_l = 15mm

Thickness of lever h_l = 45 mm ($\because h_l = 3b_l$)

Example 6.28

A single band brake operates on a drum 600 mm in diameter that is running at 200 rpm while absorbing 15 kW of power. The coefficient of friction is 0.25. The brake band has a contact of 270° and one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and is placed perpendicular to the diameter that bisects the angle of contact. Determine

i) Maximum effort required to stop the rotation of drum.

ii) Width of 2.5 mm thick steel band, if the maximum tensile stress in it is not to exceed 56 MPa.

iii) Design the lever **VTU Jan/Feb, 2004**

Data :

$D = 600$ mm $\therefore$ $R = 300$ mm; $N = 15$ kW; $n = 200$ rpm; $\mu = 0.25$; $\theta = 270°$; $h = 2.5$ mm; $\sigma_d = 56$ MPa

Solution :

i) Maximum effort

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{15}{200} = 716250$ Nmm

Also $M_t = F_\theta . R$

i.e., $716250 = F_\theta \times 300$

$\therefore$ Tangential force $F_\theta = 2387.5$ N

For cw rotation, actuating force $F = \frac{F_\theta . b}{a}\left[\frac{1}{e^{u\theta}-1}\right]$ ---- 19.151 (DDHB)

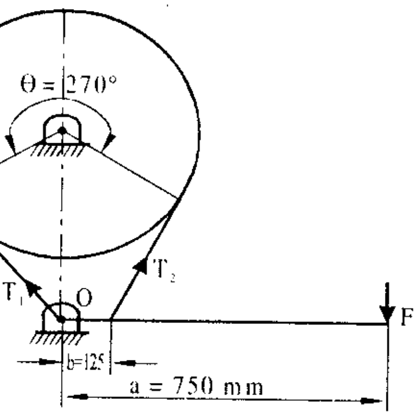


Fig. 6.29

$$= \frac{2387.5 \times 125}{750}\left[\frac{1}{e^{0.25 \times 270 \times \pi/180} - 1}\right] = 177\ N$$

For ccw rotation, actuating force $F = \frac{F_0 . b}{a}\left[\frac{e^{u\theta}}{e^{u\theta} - 1}\right]$ ---- 19.152 (DDHB)

$$= \frac{2387.5 \times 125}{750}\left[\frac{e^{0.25 \times 270 \times \pi/180}}{e^{0.25 \times 270 \times \pi/180} - 1}\right] = 574.9\ N$$

∴ Maximum effort required to stop the motion $F_{max} = 574.9\ N$

ii) Width of steel band

Width of steel band $w = \frac{T_1}{h\sigma_d}$ ---- 19.161 (DDHB)

$M_t = (T_1 - T_2)\,R$

i.e., $716250 = (T_1 - T_2)\,300 \quad \therefore\ T_1 - T_2 = 2387.5\ N$ ---- (i)

$\frac{T_1}{T_2} = e^{u\theta} = e^{0.25 \times 270 \times \pi/180} = 3.2482 \quad \therefore\ T_1 = 3.2482\ T_2$ ---- (ii)

Substituting (ii) in (i)

$3.2482.\ T_2 - T_2 = 2387.5 \quad \therefore\ T_2 = 1061.96\ N$ and $T_1 = 3449.46\ N$

$\therefore\ w = \frac{3449.46}{2.5 \times 56} = 24.69\ mm$

take, Width of band $w = 25\ mm$

Assume 4 rivets of 8 mm size is used and these are arranged in two rows. Hence the band will not be weakened by more than two rivet holes

Considering the rivet holes

$$\text{Width of band } w = 25 + 2 \times \text{Diameter of rivet hole}$$
$$= 25 + 2 \times 9 = 43 \text{ mm}$$
$$\therefore \text{ Adopt width of band } w = 45 \text{ mm}$$

Check for the stresses

$$\text{Maximum shear stress in the rivet} = \frac{T_1}{i\frac{\pi}{4}d^2} = \frac{3449.46}{4 \times \frac{\pi}{4} \times 8^2} = 17.156 \text{ N/mm}^2$$

$$\text{Maximum crushing stress in the rivet} = \frac{T_1}{idh} = \frac{3449.46}{4 \times 8 \times 2.5} = 43.12 \text{ N/mm}^2$$

where i = Number of rivets and d = Diameter of rivet. These stresses are within limits.

iii) Design of lever

Assume C40 steel as lever material and Fos = 3

From Table 1.5 (Old DDHB) for C40 steel

$$\sigma_y = 328.6 \text{ MPa} \quad \therefore \sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{3} = 109.53 \text{ N/mm}^2$$

Considering the lever as a cantilever and neglecting the effect of T_1 or T_2

$$\text{Maximum Bending Moment } M_b = F_{max}.a = 574.9 \times 750 = 431175 \text{ Nmm}$$

$$\text{We know } \frac{M_b}{I} = \frac{\sigma_b}{c} \text{ where } I = \frac{b_1 h_1^3}{12} \text{ and } c = \frac{h_1}{2}$$

$$\text{Assume } h_1 = 3b_1$$

$$\therefore I = \frac{b_1(3b_1)^3}{12} = \frac{27}{12}b_1^4 \text{ and } c = \frac{3b_1}{2} = 1.5b_1$$

$$\therefore \frac{431175}{\frac{27}{12}b_1^4} = \frac{109.53}{1.5b_1} \quad \therefore b_1 = 13.79 \text{ mm}$$

take, Width of lever $b_1 = 14$ mm

$\therefore$ Thickness of lever $h_1 = 42$ mm $\quad (\because h_1 = 3b_1)$

Example 6.29

The band brake of a crane is actuated by a lever, the free end of which is pulled upward in order to apply the brake. The length of lever = 440 mm. The tight end of the band is attached to the fulcrum of the lever and the slack end to a pin 50 mm from fulcrum i.e. 390 mm from the free end of the lever where pull is applied. The diameter of the brake drum = 1000 mm and that of barrel = 650 mm. Find the necessary pull at the end of the brake lever in order to hold a load of 20 kN. Take $\mu = 0.35$ and angle of lap = 300°.

Data :

$$\mathbf{D = 1000 \text{ mm} \therefore R = 500 \text{ mm}; \mu = 0.35; \theta = 300°}$$

Diameter of barrel = 650 mm; Load on the barrel = 20 kN = 20×10^3 N

Solution :

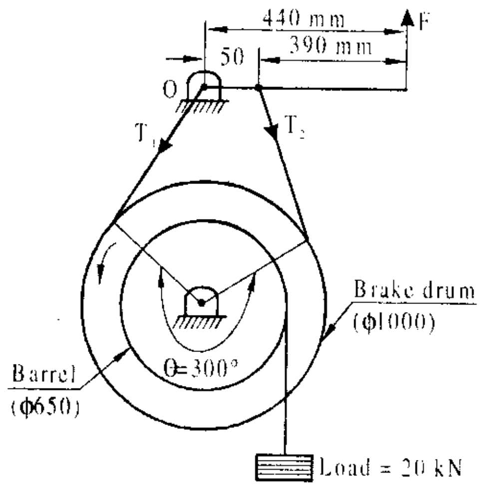


Fig. 6.30

The arrangement of the given band brake is as shown in Fig. 6.30. Since the tight end of the band is attached to the fulcrum, the direction of rotation of the brake drum is ccw

Now, Torque on the barrel shaft $= 20 \times 10^3 \times \dfrac{650}{2} = 65 \times 10^5$ Nmm. Since the barrel and the brake drum are on the same shaft, braking torque $M_t = 65 \times 10^5$ Nmm

$$\text{Also } M_t = (T_1 - T_2)R$$

$$\text{i.e. } 65 \times 10^5 = (T_1 - T_2)500$$

$$\therefore T_1 - T_2 = 13000\text{N} \qquad \text{-----(i)}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.35 \times 300 \times \pi/180} = 6.25$$

$$\therefore T_1 = 6.25\, T_2 \qquad \text{-----(ii)}$$

Substituting in equation (i)

$$6.25\, T_2 - T_2 = 13000$$

$$\therefore T_2 = 2476.2\text{ N and } T_1 = 15476.2\text{ N}$$

Taking moments about O

$$F \times 440 = T_2 \times 50$$

$$\text{i.e., } F \times 440 = 2476.2 \times 50$$

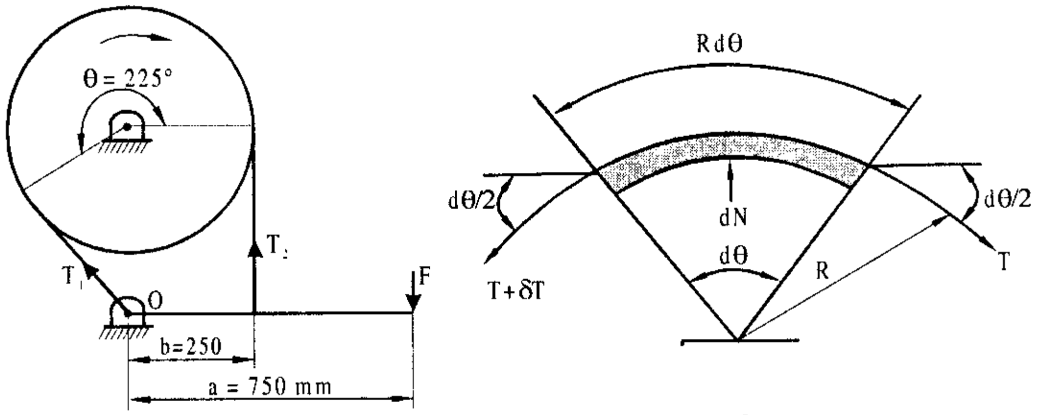
∴ Force required to hold the load F = 281.4 N

Example 6.30

A simple band brake is shown in Fig. 6.31(a) and assume the following data : b = 250 mm; a = 750mm; θ = 225°; R = 250 mm. The width of the friction lining is 60 mm and the coefficient of friction is 0.4. The maximum intensity of pressure is 0.25 N/mm². Calculate :

(i) Band tensions on tight and loose sides

(ii) Actuating force (iii) Torque capacity of the brake



(a) *Fig. 6.31* *(b)*

Data :

$$\theta = 225°;\ R = 250\text{ mm};\ w = 60\text{ mm};\ \mu = 0.4;\ p_{max} = 0.25\text{ N/mm}^2$$

Solution :

i) Band tensions on tight and loose sides

The intensity of pressure will be maximum when the band tension is equal to T_1

$$\text{i.e., } p_{max} = \frac{T_1}{R.w}$$

$$\text{i.e., } 0.25 = \frac{T_1}{250 \times 60}$$

∴ Tension on tight side

$$T_1 = 3750\text{N}$$

$$\text{Also } \frac{T_1}{T_2} = e^{\mu\theta}$$

$$\text{i.e., } \frac{3750}{T_2} = e^{0.4 \times 225 \times \pi/180}$$

∴ Tension on slack side $T_2 = 779.55$ N

ii) Actuating force

Taking moments about O

$$F \times 750 = T_2 \times 250$$

$$\text{i.e., } F \times 750 = 779.55 \times 250$$

∴ Actuating force $F = 259.85$ N

iii) Torque capacity of the brake

Torque $M_t = (T_1 - T_2) R = (3750 - 779.55)\, 250$

$= 742612.5$ Nmm $= 742.6125$ Nm

Note:

An element of the band subtending an angle dθ is shown in Fig 6.31(b). The elemental area of the friction lining = Rdθw, where w = width of the lining. Let T and T + δT be the tensions in the band in the loose and tight sides respectively. If p is the intensity of pressure, then normal reaction dN = p Rwdθ --- (i)

Considering equilibrium of vertical forces on the element

$$dN = T \sin\frac{d\theta}{2} + (T + \delta T)\sin\frac{d\theta}{2}$$

$$\text{For small angles } \sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$$

$$\therefore dN = T\frac{d\theta}{2} + (T + \delta T)\frac{d\theta}{2} = Td\theta \quad \text{--- (ii)}$$

(Neglect $\delta T . \frac{d\theta}{2}$ since very small)

Equating (i) and (ii)

$$Td\theta = pRwd\theta;\ \therefore\ p = \frac{T}{Rw}.$$

$$\text{For maximum intensity of pressure } P_{max} = \frac{T_1}{R.w}$$

Example 6.31

Fig. 6.32 shows a simple band brake which is applied to a shaft carrying a flywheel [i.e., rotating drum] of mass 200 kg and of radius of gyration 150 mm. The flywheel rotates at 200 rpm. The brake drum diameter is 250 mm and coefficient of friction is 0.20. The angle of lap of the band on the drum is 210°. If the braking torque is 40 Nm find (i) Actuating force (ii) Number of turns of the flywheel before it comes to rest (iii) Time taken by the flywheel to come to rest.

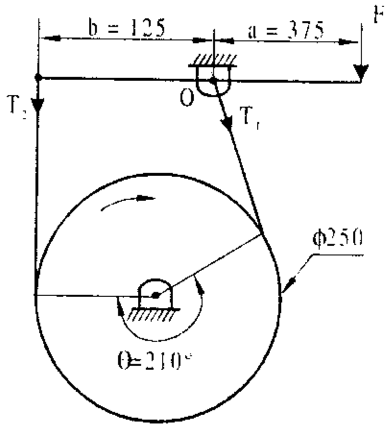


Fig. 6.32

Data :

Mass of flywheel M =200 kg; k = 150 mm = 0.15 m; n = 200 rpm; D = 250 mm
∴ R = 125 mm; μ = 0.2; θ = 210°; M_t = 40 Nm = 40 × 10^3 Nmm

Solution :

i. Actuating force

$$\text{Torque } M_t = (T_1 - T_2)R$$
$$\text{i.e., } 40\times10^3 = (T_1 - T_2)\times125$$
$$\therefore\ T_1 - T_2 = 320 \qquad \text{----(i)}$$
$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.2\times210\times\pi/180} = 2.0814;\ \therefore\ T_1 = 2.0814\ T_2 \qquad \text{----(iii)}$$

Substituting in equation (i)

$$2.0814\,T_2 - T_2 = 320$$
$$\therefore\ T_2 = 295.9\ \text{N and } T_1 = 615.9\ \text{N}$$

Taking moment about O

$$F\times375 = T_2\times125$$
$$\text{i.e., } F\times375 = 295.9\times125$$
$$\therefore\ \text{Actuating force } F = 98.63\ \text{N}$$

ii) Number of turns of the flywheel before it comes to rest

The kinetic energy of the rotation of the flywheel is used to overcome the workdone due to braking torque before the flywheel comes to rest.

K.E due to rotation $= \frac{1}{2} I\omega^2$ where $I = Mk^2$ and $\omega = \frac{2\pi n}{60}$

$$= \frac{1}{2}(200 \times 0.15^2)\left(\frac{2\pi 200}{60}\right)^2 = 986.96\text{ Nm}$$

Work done by the braking torque in 'i' number of turns of flywheel

$= M_t \times$ Angular displacement in i turns

$= 40 \times 2\pi i = 80\pi i$

Since KE of flywheel = Work done by braking torque

i.e., $986.96 = 80\pi i \therefore i = 3.927$

iii) Time taken by the flywheel to come to rest after applying the brake

M.I. of flywheel $I = Mk^2 = 200 \times 0.15^2 = 4.5\text{ kgm}^2$

Angular acceleration of flywheel $\alpha = \frac{M_t}{I} = \frac{40}{4.5} = 8.889\text{ rad/sec}^2$

Angular velocity of flywheel $\omega = \frac{2\pi n}{60} = \frac{2\pi 200}{60} = 20.944\text{ rad/sec}$

Now $\alpha t = \omega \therefore t = \frac{\omega}{\alpha} = \frac{20.944}{8.889} = 2.3562\text{ secs}$

$\therefore$ Time required to bring the wheel to rest = 2.3562 sec.

Example 6.32

A simple band brake has a drum diameter of 500 mm. A vertical passing through the centre of drum also passes through the fulcrum which is 350 mm below the centre of drum. The length of brake lever is 800 mm. One end of the band is attached to the brake lever at a distance of 250 mm from the fulcrum end and the other end is attached to the fulcrum. Power absorbed is 30 kW at a rated speed of 1000 rpm. Selecting suitable materials and assuming an appropriate value for the factor of safety, determine

i) Dimensions of rectangular cross-section of the brake lever assuming that the depth is twice the width.

ii) Dimensions of the rectangular cross section of the band assuming that the width is ten times the depth **(BU, Feb. 97 VTU, Jan/Feb 2005)**

Data :

$$D = 500\text{ mm} \therefore R = 250\text{ mm};\ N = 30\text{ kW};\ n = 1000\text{ rpm};\ h_l = 2b_l;\ w = 10\,h$$

Solution :

i) Dimensions of rectangular c/s of brake lever

Braking torque $M_t = 9550 \times 1000 \times \frac{N}{n}$

$$= 9550 \times 1000 \times \frac{30}{1000} = 286500\text{ Nmm}$$

Also $M_t = F_\theta . R$; $\therefore 286500 = F_\theta \times 250$

$\therefore$ Tangential force $F\theta = 1146\text{ N}$

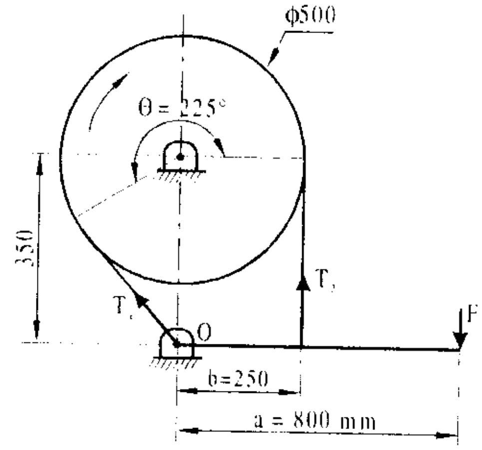


Fig. 4.31

Draw the Fig. 6.33 with a suitable scale [i.e., scale 1 : 10] and measure angle of lap θ from the diagram i.e., θ = 225°.

Now for cw rotation $F = F_\theta \cdot \frac{b}{a}\left[\frac{1}{e^{\mu\theta}-1}\right]$ Assume $\mu = 0.3$ ---- 19.151

$$= 1146 \times \frac{250}{800}\frac{1}{\left[e^{0.3\times225\times\pi/180}-1\right]} = 159.3\text{ N}$$

For ccw rotation $F = F_\theta \cdot \frac{b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right]$ ---- 19.152

$$= 1146 \times \frac{250}{800}\left[\frac{3.2482}{3.2482-1}\right] = 517.4\text{ N}\left(\because e^{\mu\theta} = e^{0.3\times225\times\frac{\pi}{180}} = 3.2482\right)$$

∴ Maximum actuating force $F_{max} = 517.4$ N

Neglecting the effect of T_1 or T_2, maximum bending moment $M_b = F_{max}.a = 517.4 \times 800 = 413920$N

We know $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_l h_l^3}{12} = \frac{b_l(2b_l)^3}{12} = \frac{2}{3}b_l^4$ and

$$c = \frac{h_l}{2} = \frac{2b_l}{2} = b_l$$

Assume C40 steel as lever material and FOS = 3

∴ From Table 1.5 (Old DDHB) for C40 steel $\sigma_y = 328.6$ MPa

$$\sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{3} = 109.53\text{ N/mm}^2 = \sigma_b$$

$$\therefore \frac{413920}{\frac{2}{3}b_l^4} = \frac{109.53}{b_l}\text{; i.e., } b_l = 17.83\text{ mm}$$

Take width of brake lever $b_1 = 18$ mm

$\therefore$ thickness of brake lever $h_1 = 36$ mm ($\because h_1 = 2b_1$)

iv) Dimensions of rectangular c/s of band

Thickness of band $h = 0.005D$ ---- 19.160

$= 0.005 \times 500 = 2.5$ mm

$\therefore$ Width of band $w = 10\,h = 10 \times 2.5 = 25$ mm

6.23 DIFFERENTIAL BAND BRAKE

Fig. 6.34 shows a differential band brake. In differential band brake no end of the band is connected to the fulcrum. The two ends of the band are attached to pins on the lever at a distance of b_1 and b_2 from the fulcrum. The force F will act in the upward direction when $b_2 > b_1$ and it will act in the downward direction when $b_2 < b_1$

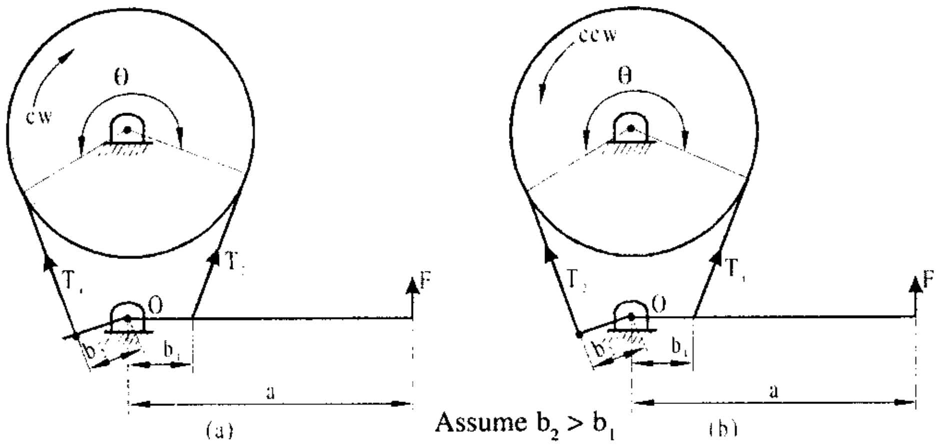


Fig. 6.34

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n}$ where M_t in Nmm

i) Consider cw rotation (Refer Fig. 6.34a)

Method I

Braking torque $M_t = (T_1 - T_2)\,R$

Also $\frac{T_1}{T_2} = e^{\mu\theta}$ where θ is the angle of lap in radians. Using the above two relations T_1 and T_2 can be calculated.

Now taking moments about O

$$F \times a + T_2 \times b_1 = T_1 \times b_2$$

$\therefore$ Actuating force $F = \frac{T_1 b_2 - T_2 b_1}{a}$

Method II

Braking torque $M_t = F_\theta . R$

Using the above relation F_θ can be calculated.

$$\text{Also } F_\theta = T_1 - T_2 = T_2\left[\frac{T_1}{T_2} - 1\right] = T_2\,[e^{u\theta} - 1]$$

$$\therefore\ T_2 = \frac{F_\theta}{e^{u\theta} - 1}$$

Taking moments about O [Refer Fig. 6.34a]

$$F \times a + T_2 \times b_1 = T_1 \times b_2$$

$$\therefore\ F = \frac{T_1 b_2 - T_2 b_1}{a} = \frac{T_2\left(\frac{T_1}{T_2} b_2 - b_1\right)}{a}$$

$$= \frac{F_\theta}{(e^{u\theta} - 1)}\left(\frac{e^{u\theta}.b_2 - b_1}{a}\right)$$

$$\therefore\ \text{Actuating force } F = \frac{F_\theta}{a}\left(\frac{e^{u\theta}.b_2 - b_1}{e^{u\theta} - 1}\right) \qquad \text{---- 19.156}$$

For self locking, the force F = 0

$$\therefore\ e^{u\theta} b_2 \le b_1$$

$$\text{i.e., } e^{u\theta} \le \frac{b_1}{b_2}$$

ii) Consider ccw rotation (Refer Fig. 6.34b)

Method I

Braking torque $M_t = (T_1 - T_2) R$

$$\text{Also } \frac{T_1}{T_2} = e^{u\theta}$$

Using the above two relations T_1 and T_2 can be calculated

Now taking moments about O

$$F \times a + T_1 \times b_1 = T_2 \times b_2$$

$$\therefore\ \text{Actuating force} F = \frac{T_2 b_2 - T_1 b_1}{a}$$

Method II

Braking torque $M_t = F_\theta . R$

Using the above relation F_θ can be calculated

$$\text{Also } F_\theta = T_1 - T_2 = T_1\left(1-\frac{T_2}{T_1}\right) = T_1\left[1-\frac{1}{e^{u\theta}}\right] = T_1\left(\frac{e^{u\theta}-1}{e^{u\theta}}\right)$$

$$\therefore\ T_1 = \frac{F_\theta . e^{u\theta}}{e^{u\theta}-1}$$

Taking moments about O [Refer Fig. 6.34b]

$$F \times a + T_1 b_1 = T_2 b_2$$

$$\therefore\ F = \frac{T_2 b_2 - T_1 b_1}{a} = \frac{T_1\left(\frac{T_2}{T_1} b_2 - b_1\right)}{a}$$

$$= \frac{F_\theta . e^{u\theta}}{e^{u\theta}-1}\left(\frac{\frac{1}{e^{u\theta}} . b_2 - b_1}{a}\right) = \frac{F_\theta\left(b_2 - e^{u\theta} b_1\right)}{\left(e^{u\theta}-1\right) . a}$$

$$\therefore\ \text{Actuating force} F = \frac{F_\theta}{a}\left[\frac{b_2 - b_1 e^{u\theta}}{e^{u\theta}-1}\right] \qquad \text{---- 19.157}$$

For self locking $F = 0$

$$\therefore\ b_2 \le b_1\, e^{u\theta}$$

i.e., $$\frac{b_2}{b_1} \le e^{u\theta}$$

Note :

If the direction of F is downward, use the clockwise formula for ccw rotation and the counter clockwise formula for cw rotation from the design data hand book.

Example 6.33

A differential band brake has an operating lever 225 mm long. The ends of the brake band are attached so that their operating arms are 38 mm and 127 mm long. Brake drum diameter = 600 mm Arc of contact is 300° and μ = 0.22. The band is 3.2 mm × 100 mm

i) Find the least force required at the end of operating lever when the band is subjected to a stress of 55 N/mm².

ii) What is the torque applied to the brake drum shaft.

iii) Is this brake self locking? Prove your answer. (VTU, Dec 06/Jan 07)

Data :

D = 600 mm; ∴ R = 300 mm; μ = 0.22; θ = 300°; h = 3.2 mm; w = 100 mm; σ_d = 55 N/mm²

Solution :

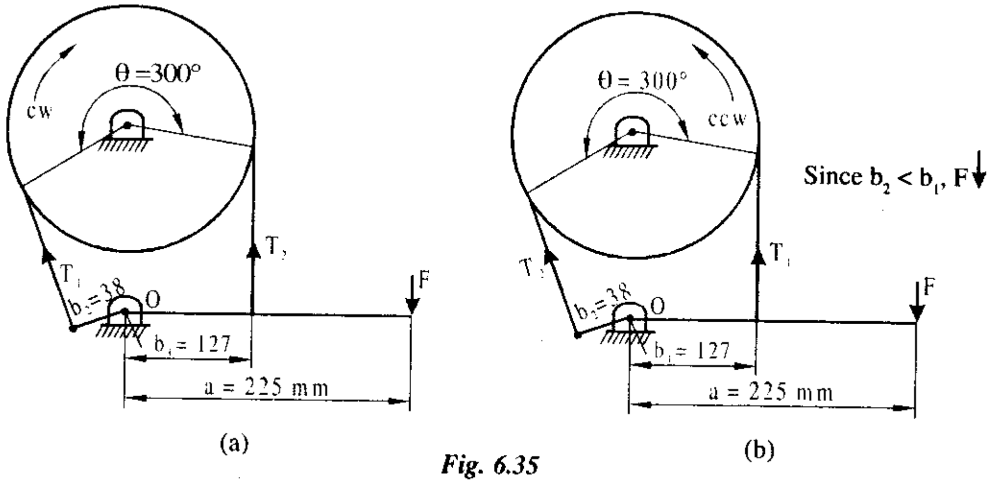


Fig. 6.35

i) Least actuating force

$$T_1 = \sigma_d wh = 55 \times 100 \times 3.2 = 17600\,\text{N} \qquad \text{---- 19.161}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.22 \times 300 \times \frac{\pi}{180}} = 3.16425$$

$$\therefore\ T_2 = \frac{T_1}{3.1643} = \frac{17600}{3.16425} = 5562.1\,\text{N}$$

Taking moments about O for cw (Refer Fig. 6.35 a)

$$F \times a + T_1 \times b_2 = T_2 \times b_1$$

i.e., $F \times 225 + 17600 \times 38 = 5562.1 \times 127$

$\therefore$ Actuating force $F = 167.1\,\text{N}$

Taking moments about O for ccw rotation [Refer Fig. 6.35 b]

$$F \times a + T_2 b_2 = T_1 \times b_1$$

i.e., $F \times 225 + 5562.1 \times 38 = 17600 \times 127$

i.e., $F = 8994.8\,\text{N}$

$\therefore$ Least actuating force $F = 167.1\,\text{N}$

ii) Braking torque

Braking torque $M_t = (T_1 - T_2)R = [17600 - 5562.1] \times 300$

$= 3611370\,\text{N mm} = 3611.37\,\text{Nm}$

iii) Check for self locking

For cw rotation

$$F = \frac{T_2 b_1 - T_1 b_2}{a}$$

For self locking

$$T_2\, b_1 \le T_1\, b_2$$

$$\text{i.e.,}\quad \frac{b_1}{b_2} \le \frac{T_1}{T_2}$$

$$\text{i.e.,}\quad \frac{127}{38} \le 3.16425$$

$$\text{i.e.,}\quad 3.3421 \le 3.16425$$

But $3.3421 > 3.16425$, hence no self locking in cw rotation.

For ccw rotation

$$F = \frac{T_1 b_1 - T_2 b_2}{a}$$

For self locking

$$T_1 b_1 \le T_2 b_2$$

$$\text{i.e.,}\quad \frac{T_1}{T_2} \le \frac{b_2}{b_1}$$

$$\text{i.e.,}\quad 3.16425 \le \frac{38}{127}$$

$$\le 0.2992$$

But $3.16425 > 0.2992$, hence no self locking in ccw rotation.

Example 6.34

The torque absorbed in the band brake shown in Fig. 6.36 is 400×10^3 Nmm. Design the band and lever, taking $\mu = 0.27$ and diameter of drum as 400 mm. The allowable stress in band may be taken as 70 N/mm^2.

VTU, August 2001

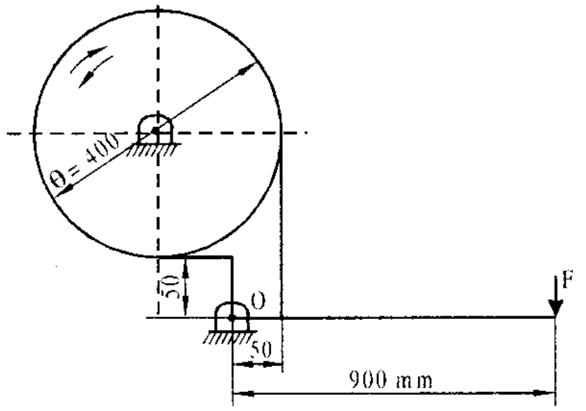


Fig. 6.36

Data:

$M_t = 400 \times 10^3$ Nmm; $\mu = 0.27$; $D = 400$ mm $\therefore$ $R = 200$ mm; $\sigma_d = 70$ N/mm^2; $\theta = 270°$ (refer figure)

Solution :

The brake shown in Fig. 6.36 is a two way band brake. It is so designed that it can operate equally well in both clockwise and counter clockwise rotation of the brake drum. This is possible since both the moments of the tension T_1 and T_2 act in the same direction and in the opposite direction to the moment of the operating force. The moment arms of both the tensions are equal.

i) Design of band

Braking torque $M_t = (T_1 - T_2) R$

i.e., $400 \times 10^3 = (T_1 - T_2) 200$

$\therefore \; T_1 - T_2 = 2000 \text{ N}$ ---- (i)

Also $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.27 \times 270 \times \frac{\pi}{180}} = 3.5692$

i.e., $T_1 = 3.5692\, T_2$ ----(ii)

Substituting in equation (i)

$3.5692\, T_2 - T_2 = 2000$

$\therefore \; T_2 = 778.45 \text{ N}$ and $T_1 = 2778.45 \text{ N}$

thickness of band $h = 0.005\, D = 0.005 \times 400 = 2$ mm. ---- 19.160

Width of band $w = \frac{T_1}{h\sigma_d} = \frac{2778.45}{2 \times 70} = 19.846 \text{ mm} \approx 20 \text{ mm}$ ---- 19.161

Assume 4 rivets of 8 mm size is used and these are arranged in two rows. Hence the band will not be weakened by more than two rivet holes.

Considering the rivet holes, width of band w = 20 + 2 × diameter of rivet hole = 20 + 2 × 9 = 38 mm

$\therefore$ Adopt width of band $w = 40$ mm

Check for the stresses

Maximum shear stress in the rivet $= \frac{T_1}{\text{Number of rivets} \times \text{Area of c/s of rivet}} = \frac{2778.45}{4 \times \frac{\pi}{4} \times 8^2}$

$= 13.82 \text{ N/mm}^2$

Maximum crushing stress in the rivet $= \frac{T_1}{idh} = \frac{2778.45}{4 \times 8 \times 2} = 43.41 \text{ N/mm}^2$

Where i = number of rivets d = diameter of rivet

These stresses are within limits.

ii) Design of lever

Taking moment about O for cw

$F \times 900 = T_1 \times 50 + T_2 \times 50 = 50\,[2778.45 + 778.45]$

$\therefore$ Actuating force $F = 197.6$ N

Assume rectangular cross-section for the lever with depth equal to thrice the width.

Since the brake is to operate for both the directions of rotation, the shorter arm is to be designed for maximum bending moment.

$\therefore$ $M_b = T_1 \times 50 = 2778.45 \times 50 = 138922.5$ Nmm. Assume the lever is made of C40 steel with factor of safety = 3.

From Table 1.5 (Old DDHB) for C40 steel, $\sigma_y = 328.6$ MPa

$$\therefore \sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{3} = 109.53 \text{ N/mm}^2 = \sigma_b$$

We have $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_1 h_1^3}{12} = \frac{b_1(3b_1)^3}{12} = \frac{27}{12} b_1^3$ and $c = \frac{h_1}{2} = \frac{3b_1}{2} = 1.5 b_1$

$$\therefore \frac{138922.5}{\frac{27}{12} b_1^4} = \frac{109.53}{1.5 b_1} \quad \text{i.e., } b_1 = 9.456 \text{ mm}$$

Adopt width of lever $b_1 = 10$mm

$\therefore$ Thickness of lever $h_1 = 30$ mm ($\because h_1 = 3b_1$)

Example 6.35

A differential band brake shown in Fig. 6.37 (a) operates on a drum diameter of 500 mm. The drum rotates at 300 rpm in counter clockwise direction and absorbs 36 kW. $\mu = 0.25$. Determine

(i) Force F required to operate the brake.

(ii) Width of band required for this brake if thickness is 5 mm and allowable tensile stress on band material is 72 N/mm².

(iii) Design the lever if the maximum force is twice that of calculated force. Use C30 steel and FOS = 4 based on ultimate stress.

(iv) Resultant fulcrum pin reaction and pin diameter.

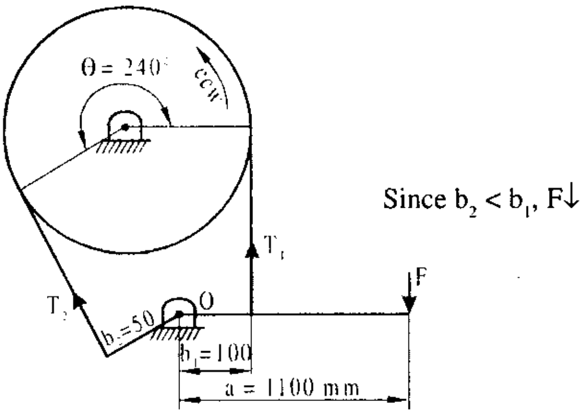


Fig. 6.37 (a)

Data :

$D = 500$ $\therefore$ $R = 250$ mm; $n = 300$ rpm; $N = 36$ kW $\mu = 0.25$; $h = 5$ mm; $\sigma_d = 72$ N/mm²
Lever material C30 steel, FOS based on ultimate strength = 4.

Solution :

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{36}{300} = 1146000\,Nmm$

i) Actuating force

$$\text{Braking torque } M_t = (T_1 - T_2)R$$

$$\text{i.e., } 1146000 = (T_1 - T_2)250$$

$$\therefore T_1 - T_2 = 4584\,N \quad \text{----(i)}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times \frac{\pi}{180} \times 240} = 2.85$$

$$\text{i.e., } T_1 = 2.85\,T_2 \quad \text{----(ii)}$$

Substituting in equation (i)

$$2.85\,T_2 - T_2 = 4584$$

$$\therefore T_2 = 2477.83\,N \text{ and } T_1 = 7061.83\,N$$

Taking moments about O for ccw direction

$$F \times a + T_2 \times b_2 = T_1 \times b_1$$

$$\text{i.e., } F \times 1100 + 2477.83 \times 50 = 7061.83 \times 100$$

$$\therefore \text{Actuating force } F = 529.35\,N$$

ii) Design of band

$$\text{Thickness of band } h = 5\,mm \text{ (given)}$$

$$\text{Width of band } w = \frac{T_1}{h\sigma_d} = \frac{7061.83}{5 \times 72} = 19.62\,mm \approx 20\,mm \quad \text{---- 19.161}$$

Assume 4 rivets of 8 mm size is used and these are arranged in two rows. Hence the band will not be weakened by more than two rivet holes.

Considering the rivet holes

$$\text{Width of band } w = 20 + 2 \times \text{diameter of rivet hole} = 20 + 2 \times 9 = 38\,mm$$

$\therefore$ Adopt width of band $w = 40$ mm.

Check for the stresses

$$\text{Maximum shear stress in the rivet} = \frac{T_1}{i\frac{\pi}{4}d^2} = \frac{7061.83}{4 \times \frac{\pi}{4} \times 8^2} = 35.12\,N/mm^2$$

$$\text{Maximum crushing stress in the rivet} = \frac{T_1}{idh} = \frac{7061.83}{4 \times 8 \times 5} = 44.136\,N/mm^2$$

Where i = number of rivets; d = diameter of rivet.

These stresses are within limits.

iii) Design of lever

Maximum bending moment on the lever will be at the fulcrum pin. Considering the shorter arm

$$M_b = 2T_2 \times 50 \quad (\because \text{ Maximum force is twice that of calculated force})$$

$$= 2 \times 2477.83 \times 50 = 247783\,Nmm$$

From Table 1.5 (Old DDHB) for C30 steel, σ_u = 490.4 to 588.4 MPa

$$\text{take } \sigma_u = 540 \text{ MPa} \;\therefore\; \sigma_b = \frac{\sigma_u}{FOS} = \frac{540}{4} = 135 \text{ N/mm}^2$$

Assume rectangular cross-section for the lever with depth equal to thrice the width

$$\therefore\; I = \frac{b_1 h_1^3}{12} = \frac{b_1 (3b_1)^3}{12} = \frac{27}{12} b_1^4;\; c = \frac{h_1}{2} = \frac{3b_1}{2} = 1.5\, b_1$$

$$\text{We have } \frac{M_b}{I} = \frac{\sigma_b}{c}$$

$$\text{i.e., } \frac{247783}{\frac{27}{12} b_1^4} = \frac{135}{1.5 b_1} \quad \therefore\; b_1 = 10.69 \text{ mm}$$

Adopt width of lever $b_1 = 12$ mm

Thickness of lever h_1 = 36 mm ($\because$ $h_1 = 3\, b_1$)

iv) Resultant fulcrum pin reaction and pin diameter

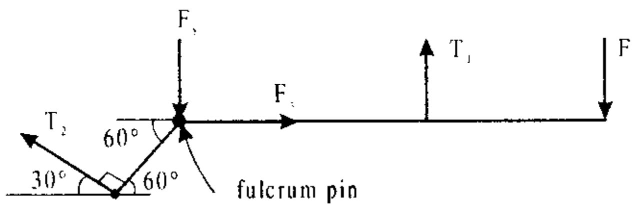


Fig. 6.37 (b) : Free body diagram of brake lever

For the equilibrium of lever, sum of vertical and horizontal forces must be equal to zero.

Considering the vertical forces.

$$F - T_1 + F_y - T_2 \sin 30 = 0$$

$$\text{i.e., } 529.35 - 7061.83 + F_y - 2477.83 \sin 30 = 0$$

$$\therefore\; F_y = 7771.4 \text{ N}$$

Considering the horizontal forces

$$F_x = T_2 \cos 30 = 2477.83 \cos 30 = 2145.86 \text{ N}$$

$$\therefore\; \text{Resultant pin reaction } F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{2145.86^2 + 7771.4^2} = 8062.22 \text{ N}$$

Since the pin is subjected to double shear

$$F_R = 2 \times \frac{\pi}{4} d_p^2 \times \tau_p \quad \text{where } d_p = \text{Diameter of pin}$$

$$\tau_p = \text{allowable shear stress in the pin}$$

Assume the pin is made of C30 steel

$$\therefore\; \tau_p = 0.5\, \sigma = 0.5 \times 135 = 67.5 \text{ N/mm}^2$$

$$\text{i.e., } 8062.22 = 2 \times \frac{\pi}{4} \times d_p^2 \times 67.5 \quad \text{i.e., } d_p = 8.72 \text{ mm}$$

$\therefore$ Adopt diameter of fulcrum pin d_p = 10 mm

Example 6.36

A band brake shown in Fig. 6.38 uses a V-belt. The pitch diameter of the V grooved pulley is 400 mm. The groove angle is 45° and the coefficient of friction is 0.3. Determine the power rating (VTU, Dec'07/Jan'08)

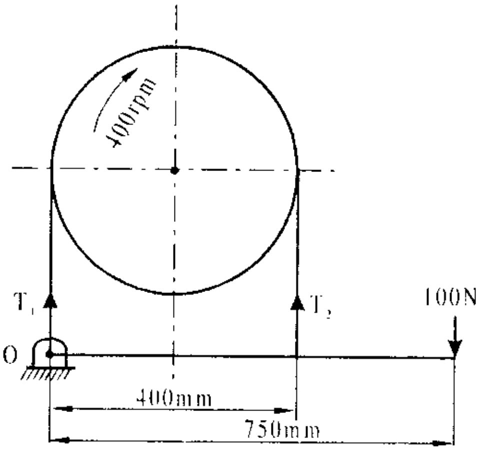


Fig. 6.38

Data :

D = 400 mm ∴ R = 200 mm; b = 400 mm; a = 750 mm; F = 100 N; n = 400 rpm; 2α = 45°;

μ = 0.3 ; θ = 180°

Solution :

Ratio of V-belt tension $\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{\frac{0.3\times180\times\pi}{180\times\sin22.5}} = 11.74$

$\therefore T_1 = 11.74\,T_2$ ---- (i)

Taking moments about 'O' for cw rotation

$100 \times 750 = T_2 \times 400$

∴ Tension on the slack side $T_2 = 187.5$ N

Substituting in equation (i)

$T_1 = 11.74 \times 187.5 = 2201.25$ N

Torque capacity of the brake $M_t = (T_1 - T_2)R = (2201.25 - 187.5)\,200$

$= 402750$ N mm $= 402.75$ Nm

Also $M_t = 9550 \times 1000 \times \frac{N}{n}$ where M_t in Nmm

ie., $402750 = 9550 \times 1000 \times \frac{N}{400}$

∴ Power capacity of the broke N = 16.87 kW

Example 6.37

A simple band brake shown in Fig. 6.39 is to be designed to stop the rotation of a shaft transmitting a power of 45 kW at a rated speed of 500 rpm. Selecting suitable materials determine,
(i) Dimensions of the rectangular cross-section of the band.
(ii) Dimensions of the rectangular cross-section of the brake lever.
(iii) Diameter of the fulcrum pin **(VTU, Jan/Feb 2006)**

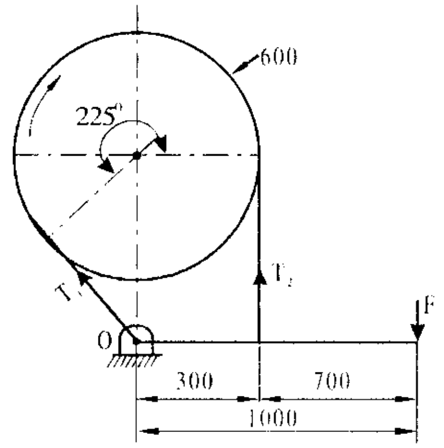


Fig. 6.39

Data :

n = 500 rpm ∴ N = 45 kW; D = 600 mm; R = 300 mm; θ = 225° ; b = 300 mm; a = 1000 mm;

Solution :

(i) Dimensions of the rectangular cross-section of the band

Torque transmitted $M_t = 9550 \times 1000 \times \frac{N}{n}$ where M_t in N mm

$= 9550 \times 1000 \times \frac{45}{500} = 85\,9500$ N mm

Also $M_t = F_0 R$

ie., $859500 = F_0 \times 300$

∴ Tangential force $F_0 = 2865$ N

Also $M_t = (T_1 - T_2) R$

ie., $859500 = (T_1 - T_2)\,300$

∴ $T_1 - T_2 = 2865$ N ---- (i)

$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 225 \times \frac{\pi}{180}} = 3.248$ Assume $\mu = 0.3$

ie., $T_1 = 3.428\,T_2$ ---- (ii)

Substituting in equation (i) 3.248 $T_2 - T_2 = 2865$

$\therefore$ Tension on the slack side of band $T_2 = 1274.4$ N

Substituting in equation (i)

Tension on the tight side of band $T_1 = 4139.4$ N

Thickness of band h = 0.005 D = 0.005 × 600 = 3 mm ---- 19.160 (DDHB)

Width of band w = $\frac{T_1}{h\sigma_d} = \frac{4139.4}{3 \times 50}$ (Assume $\sigma_d = 50$ N/mm^2) = 27.6 mm ---- 19.161

Take, Width of band w = 30 mm

Assume 4 rivets of 8 mm size is used and these are arranged in two rows. Hence the band will not be weakened by more than two rivet holes.

Considering rivet holes

Width of band w = 30 + 2 × Diameter of rivet hole

= 30 + 2 × 9 = 48 mm

$\therefore$ Adopt width of band w = 50 mm

Check for the stresses

$$\text{Maximum shear stress in the rivet} = \frac{T_1}{i\frac{\pi}{4}d^2} = \frac{4139.4}{4 \times \frac{\pi}{4} \times 8^2} = 20.6 \text{ N/mm}^2$$

$$\text{Maximum crushing stress in the rivet} = \frac{T_1}{idh} = \frac{4139.4}{4 \times 8 \times 3} = 43.12 \text{ N/mm}^2$$

These stresses are within the limit. Hence the design of the band is safe.

(ii) Dimensions of the rectangular cross-section of the brake lever

Assume C40 steel as the lever material and FOS = 3

From Table 1.5 (Old DDHB) for C40 steel

$$\sigma_y = 328.6 \text{ MPa}$$

$$\therefore \text{Permissible stress } \sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{3} = 109.53 \text{ N/mm}^2$$

For cw rotation, actuating force $F = \frac{F_0 . b}{a}\left(\frac{1}{e^{\mu\theta} - 1}\right)$ ---- 19.151 (DDHB)

$$= \frac{2865 \times 300}{1000} \frac{1}{\left(e^{0.3 \times 225 \times \frac{\pi}{180}} - 1\right)} = 382.31 \text{ N}$$

For ccw rotation, actuating force $F = F_0 \cdot \frac{b}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right)$ ---- 19.152 (DDHB)

$$= \frac{2865\times300}{1000}\times\frac{e^{0.3\times225\times\frac{\pi}{180}}}{\left(e^{0.3\times225\times\frac{\pi}{180}}-1\right)} = 1241.81\text{ N}$$

Considering the lever as a cantilever and neglecting the effect of T_1 or T_2

Maximum bending moment $M_b = F_{max} \times 1000 = 1241.81 \times 1000 = 1241810$ Nmm

Bending equation is, $\frac{M_b}{I} = \frac{\sigma_b}{c}$ where $I = \frac{b_1 h_1^3}{12}$ and $c = \frac{h_1}{2}$

Assume $h_1 = 3b_1$; $\therefore I = \frac{b_1(3b_1)^3}{12} = \frac{27}{12} b_1^4$ and $c = \frac{3b_1}{2} = 1.5b_1$

$$\therefore \frac{1241810}{\frac{27}{12}b_1^4} = \frac{109.53}{1.5b_1}\ ;\ \therefore b_1 = 19.625\text{ mm}$$

Take, Width of brake lever b_1 = 20 mm

Thickness of brake lever h_1 = 60 mm ($\because h_1 = 3b_1$)

(iii) Diameter of fulcrum pin

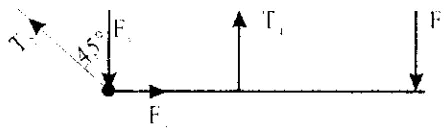


Fig. 6.40

For the equilibrium of lever, sum of vertical and horizontal forces must be equal to zero

Considering the vertical forces for ccw rotation

$T_2 \cos 45 - F_y + T_1 - F = 0$

ie., $1274.4 \cos 45 - F_y + 4139.4 - 1241.81 = 0$

$\therefore F_y = 3798.72$ N

Considering the horizontal forces for ccw rotation

$T_2 \sin 45 = F_x$

ie., $1274.4 \sin 45 = F_x$

$\therefore F_x = 901.14$ N

Resultant pin reaction for ccw rotation $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{901.14^2 + 3798.72^2} = 3904.14$ N

For cw rotations, considering vertical forces

$T_1 \cos 45 - F_y + T_2 - F = 0$

ie., $4139.4 \cos 45 - F_y + 1274.4 - 382.31 = 0$

$\therefore F_y = 3819.1$ N

Considering horizontal forces

$T_1 \sin 45 = F_x$

$4139.4 \sin 45 = F_x$

$\therefore F_x = 2927$ N

Resultant pin reaction for cw rotation $F_R = \sqrt{2927^2 + 3819.1^2} = 4811.74$ N

Considering the maximum value of pin reaction $F_{R_{Max}} = 4811.74$ N

Assume the pin is made of C40 steel

$\therefore$ Allowable shear stress in pin material $\tau_p = 0.5\,\sigma = 0.5 \times 109.53 = 54.765$ N/mm^2

Since the pin is subjected to double shear

$$F_{R_{Max}} = 2 \times \frac{\pi}{4} d_p^2 \times \tau_p \text{ where } d_p = \text{Diameter of pin}$$

$$\text{ie., } 4811.74 = 2 \times \frac{\pi}{4} \times d_p^2 \times 54.765$$

ie., $d_p = 7.48$ mm

$\therefore$ Adopt diameter of fulcrum pin $d_p = 7.5$ mm

6.24 BAND AND BLOCK BRAKE

A band and block brake is shown in Fig. 6.41 a. This brake is the modification of the band brake. It consists of a number of wooden blocks fixed inside a flexible steel band. The friction between the blocks and the drum provides braking action. Since the wooden blocks have a higher coefficient of friction, it increases the effectiveness of the brake. Also these blocks can be easily replaced if worn out.

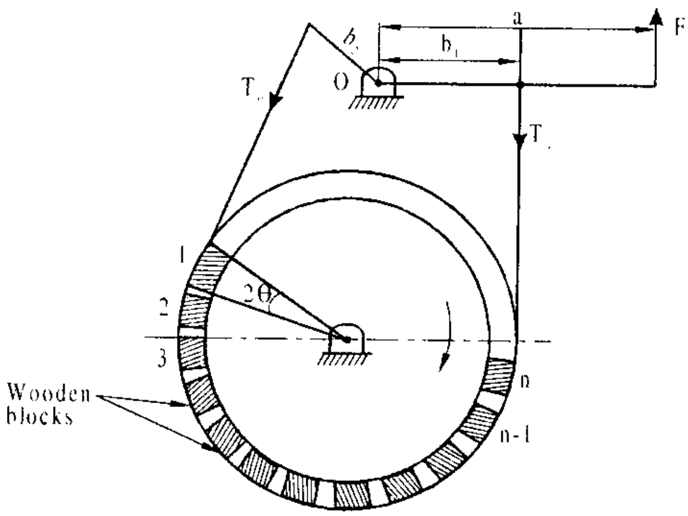


Fig. 6.41 a

If the lever is placed above the drum, then for $b_2 < b_1$, F ↑ and $b_2 > b_1$, F ↓

Let T_o = Tension on slack side

T_n = Tension on tight side after n blocks

μ = Coefficient of friction

n = Number of blocks

T_1 = Tension in the band between I[st] and 2[nd] block

T_2 = Tension in the band between 2[nd] and 3[rd] block

Consider any one block say I[st] block as shown in Fig. 6.41 b. The first block is in equilibrium under the action of the following forces

i) Tension T_o on the slack side

ii) Tension T_1 on the tight side

iii) Normal reaction R_N

iv) Friction force μR_N

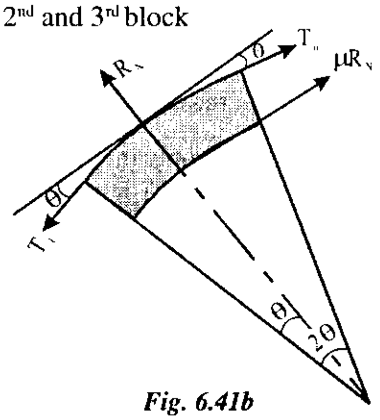


Fig. 6.41b

Resolving the forces tangentially

$$T_1 \cos\theta = T_o \cos\theta + \mu R_N$$

$$\therefore \quad \mu R_N = (T_1 - T_o)\cos\theta \qquad \text{---- (i)}$$

Resolving the forces radially

$$T_1 \sin\theta + T_o \sin\theta = R_N$$

$$\therefore \quad R_N = (T_1 + T_o)\sin\theta \qquad \text{---- (ii)}$$

Dividing equation (i) by equation (ii)

$$\frac{(T_1 - T_o)\cos\theta}{(T_1 + T_o)\sin\theta} = \frac{\mu R_N}{R_N} \quad \therefore \quad \frac{T_1 - T_o}{T_1 + T_o} = \mu \tan\theta$$

$$\text{i.e.} \quad T_1 - T_o = (T_1 + T_o)\mu\tan\theta$$

$$= T_1 \mu\tan\theta + T_o \mu\tan\theta$$

$$\text{i.e.} \quad T_1 - T_1 \mu\tan\theta = T_o + T_o \mu\tan\theta$$

$$\text{i.e.,} \quad T_1(1 - \mu\tan\theta) = T_o(1 + \mu\tan\theta)$$

$$\therefore \quad \frac{T_1}{T_o} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

$$\text{Similarly} \quad \frac{T_2}{T_1} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta} \text{ and so on}$$

$$\therefore \quad \frac{T_n}{T_o} = \frac{T_n}{T_{n-1}} \times \text{-----} \times \frac{T_3}{T_2} \times \frac{T_2}{T_1} \times \frac{T_1}{T_o}$$

$$= \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right) \times \text{-----} \times \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)\left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)\left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)$$

i.e. $\frac{T_n}{T_o} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n$ where n = Number of blocks and θ = half the angle subtended by each block on the centre of drum.

Braking torque $M_t = (T_n - T_o) R_1$ where R_1 = effective radius of drum = R + t, t = thickness of block.

Example 6.38

A band and block brake having 12 blocks, each of which subtends an angle of 15° at the drum centre. The diameter of the brake drum is 750 mm and thickness of blocks are 125 mm. The drum and the flywheel mounted on the same shaft have a mass of 1000 kg and have a combined radius of gyration of 250 mm. The two ends of the band are attached to the pins on the opposite sides of the fulcrum at a distance of 50 mm and 150 from the fulcrum. A force of 200 N is applied at a distance of 1000 mm from the fulcrum to apply the brake. The coefficient of friction between the blocks and drum may be taken as 0.3. Determine

(i) Maximum braking torque

(ii) Angular retardation of brake drum

(iii) Time taken by the system to come to rest from the rated speed of 1250 rpm.

Data :

$n = 12$; $2\theta = 15°$ $\therefore$ $\theta = 7.5°$; $D = 750$ mm $\therefore$ $R = 375$ mm; $t = 125$ mm; $M = 1000$ kg;
$k = 250$ mm $= 0.25$ m; $\mu = 0.3$; $n_o = 1250$ rpm; $F = 200$ N.

Solution :

i) Maximum braking torque

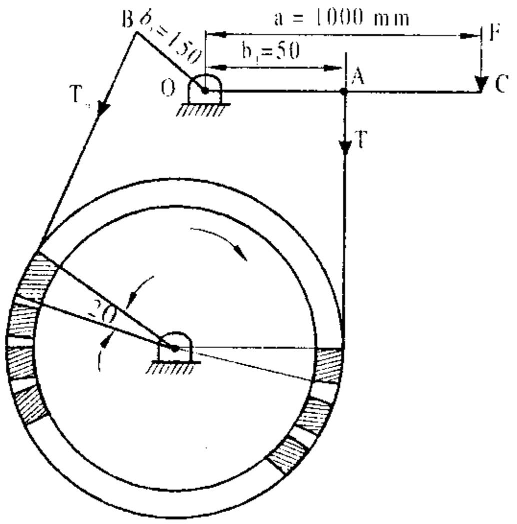


Fig. 6.42

Since $b_2 > b_1$, the force F must act downwards at C. For maximum braking torque or least force, the brake should be arranged so that the tight side of the band is attached to the shorter distance i.e., tight side of the band should be attached to A. This is possible if the drum rotates in cw direction as shown in Fig. 6.42.

$$\text{We have } \frac{T_n}{T_o} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n = \left(\frac{1+0.3\tan 7.5}{1-0.3\tan 7.5}\right)^{12} = 2.5817$$

$$\therefore\ T_n = 2.5817\,T_o$$

Now taking moments about O for cw rotation

$$F \times 1000 + T_n \times 50 = T_o \times 150$$

i.e., $200 \times 1000 + 2.5817\,T_o \times 50 = 150\,T_o$

$$\therefore\ T_o = 9562.5 \text{ N and } T_n = 24687.5 \text{ N}$$

Effective radius of drum $R_1 = R + t = 375 + 125 = 500$ mm

$\therefore$ Maximum braking torque $M_t = (T_n - T_o)\,R_1 = (24687.5 - 9562.5)\,500$

$$= 7562500 \text{ Nmm} = 7562.5 \text{ Nm}$$

ii) Angular retardation of the brake drum

Let α = angular retardation

Net torque $= I\,\alpha$ where $I = Mk^2 = 1000 \times 0.25^2 = 62.5$ kg m^2

i.e., $7562.5 = 62.5 \times \alpha$

$\therefore\ \alpha = 121$ rad/sec^2 = Maximum angular retardation

iii) Time taken by the system to come to rest

Initial angular speed $\omega_o = \dfrac{2\pi n_o}{60} = \dfrac{2\pi 1250}{60} = 130.9$ rad/sec

Final angular speed $\omega = 0$

Now $\omega = \omega_o - \alpha t$

$$\therefore\ t = \frac{\omega_0}{\alpha} = \frac{130.9}{121} = 1.082 \text{ sec}$$

Example 6.39

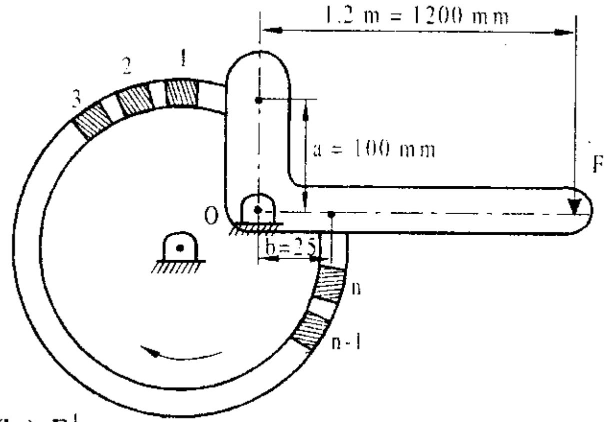
In a band and block brake $\theta = 15°$ and effective diameter is 800 mm. $\mu = 0.4$, a = 100 mm, b = 25 mm. The power absorbed at 600 rpm is 450 kW when the force applied at the end of lever at a distance of 1.20 m from the fulcrum is 200 N. Find the number of blocks.

VTU, January/February 2003

Data :

$\theta = 15°$, $D_1 = 800$ mm $\therefore$ $R_1 = 400$ mm; $\mu = 0.4$; n = 600 rpm; N = 450 kW; F = 200 N

Solution :



Since a (b_2) > b (b_1), F↓

Fig. 6.43

$$\text{Braking torque } M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{450}{600}$$

$$= 7162500 \text{ Nmm} = 7162.5 \text{ Nm}$$

$$\text{Also } M_t = (T_n - T_o) R_1$$

$$\text{i.e., } 7162500 = (T_n - T_o) 400$$

$$\therefore T_n - T_o = 17906.25 \text{ N} \qquad \text{---- (i)}$$

Assume the brake is designed for maximum torque. For maximum torque, the tight side of the band should be attached to the shorter distance and hence the drum will rotate in cw direction. Now taking moments about 0.

$$F \times 1200 + T_n \times 25 = T_o \times 100$$

$$\text{i.e., } 200 \times 1200 + T_n \times 25 = 100\, T_o$$

$$\text{i.e., } 4\,T_o - T_n = 9600 \qquad \text{---- (ii)}$$

(i) + (ii) gives

$$3\,T_o = 27506.25$$

$$\therefore T_o = 9168.75 \text{ N and } T_n = 27075 \text{ N}$$

$$\text{Now } \frac{T_n}{T_o} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n$$

$$\text{i.e., } \frac{27075}{9168.75} = \left(\frac{1 + 0.4 \tan 15}{1 - 0.4 \tan 15}\right)^n$$

$$\text{i.e., } 2.953 = (1.24)^n$$

$$n = 5.034$$

$\therefore$ Number of blocks n = 6

6.25 INTERNAL EXPANDING SHOE BRAKE

As the name implies, here the shoes are inside the brake drum. It consists of two shoes S_1 and S_2 as shown in Fig. 6.44 a. The outer surface of the shoes are lined with friction material to increase the coefficient of friction and to prevent wearing away of the metal.

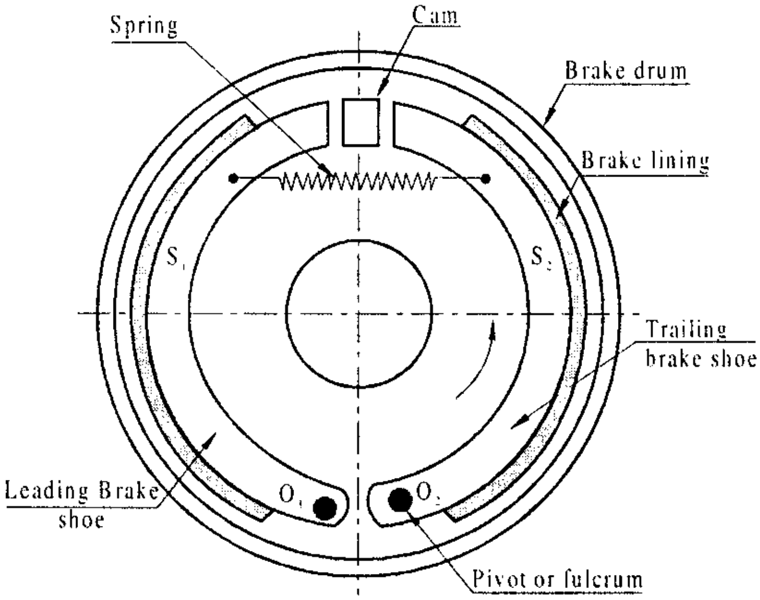


Fig. 6.44 a

Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in automobiles i.e., motor cars, trucks etc. For anticlockwise rotation of the drum, the left hand shoe is known as leading or primary shoe where as the right hand shoe is known as trailing or secondary shoe.

Let θ_1 = Angle from the pivot to start of brake lining

θ_2 = Angle from the pivot to end of brake lining

θ_a = Angle of maximum pressure

F = Actuating force

a = Distance from pivot (hinge) to centre of brake drum

c = Distance from pivot to force

b = Width of shoe

p_a = Maximum pressure on leading shoe

r = Internal radius of drum

μ = Coefficient of friction

M_t = Total braking torque
n = Speed in rpm
N = Power in kW

Consider a small element of the brake lining which subtends an angle $\delta\theta$ at the centre as shown in Fig. 6.44 b. Consider unit pressure p acting upon the element located at an angle θ from the fulcrum or pivot pin. The pressure at any point is proportional to the horizontal distance from the pivot pin and the horizontal distance is proportional to sin θ. The maximum pressure p_a is located at an angle θ_a from the pivot pin.

$$\therefore \quad \frac{p}{\sin\theta} = \frac{p_a}{\sin\theta_a}$$

$$\therefore \text{ Maximum pressure } \quad p_a = \frac{p}{\sin\theta} \cdot \sin\theta_a$$

The maximum pressure is calculated as given below

If $\theta_2 \le 90°$, then $\theta_a = \theta_2$

If $\theta_2 > 90°$, then $\theta_a = 90°$

Now normal force on the element δF_n = normal pressure × Area of the element

$$= pbrd\theta = \frac{p_a \cdot \sin\theta}{\sin\theta_a} \times br\,d\theta$$

Moment of normal force about fulcrum $\delta M_{In} = \delta F_n\, a \sin\theta = \left(\frac{p_a \sin\theta}{\sin\theta_a} brd\theta\right)(a \sin\theta)$

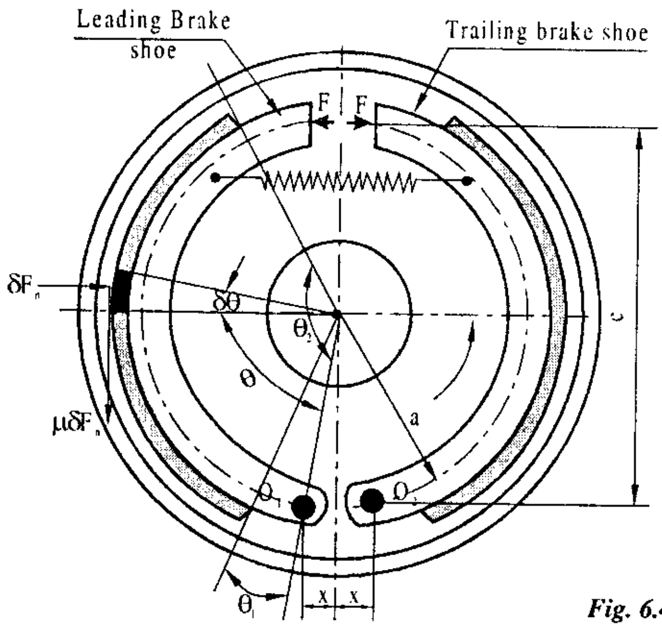


Fig. 6.44 b

$\therefore$ Moment of normal force on the shoe $M_{tn} = \int_{\theta_1}^{\theta_2} \frac{p_a bra \sin^2\theta}{\sin\theta_a} d\theta$

$$= \frac{p_a bra}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta d\theta \quad \text{--- 19.175}$$

Frictional force on the element $\delta_\mu = \mu\delta F_n = \frac{\mu p_a \sin\theta}{\sin\theta_a} \times brd\theta$

$\therefore$ Moment of frictional force about fulcrum $\delta M_{t_\mu} = (\mu\delta F_n)(r - a\cos\theta)$

$$= \left(\frac{\mu p_a \sin\theta}{\sin\theta_a} brd\theta\right)(r - a\cos\theta)$$

$\therefore$ Moment of frictional force on the shoe $M_{t\mu} = \frac{\mu p_a br}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta(r - a\cos\theta)d\theta$ ---- 19.174

Taking moments about the fulcrums O_1 for the leading shoe, $F \times c + M_{t\mu} = M_{tn}$.

$\therefore$ Actuating force $F = \frac{M_{tn} - M_{t\mu}}{c}$.

Taking moments about the fulcrum O_2 for the trailing shoe, $F \times c = M_{tn} + M_{t\mu}$

$\therefore$ Actuating force $F = \frac{M_{tn} + M_{t\mu}}{c}$ If $M_{t\mu} > M_{tn}$, then the brake becomes self locking.

Torque applied to the drum by the leading shoe $M_{t_l} = \frac{\mu p_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$ ---- 19.177

For equal actuating force on the shoes

Normal pressure on the trailing shoe $p'_a = \frac{F.c.p_a}{M_{tn} + M_{t\mu}}$

$\therefore$ Torque applied to the drum by the trailing shoe

$$M_{t_t} = \frac{\mu p'_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{M_{t_l} . p'_a}{p_a}$$

$\therefore$ Total torque applied to the drum $M_t = M_{t_l} + M_{t_\mu}$

Example 6.40

An internal expanding brake shown in Fig. 6.45 is 400 mm diameter and is actuated by a mechanism that exerts the same force F on each shoe. The shoes are identical and have a face width of 40 mm. The lining is moulded asbestos having a coefficient of friction of 0.32 and a limiting pressure of 1 MPa.
(i) Determine the actuating force
(ii) Find the braking capacity **VTU, July/August 2002**

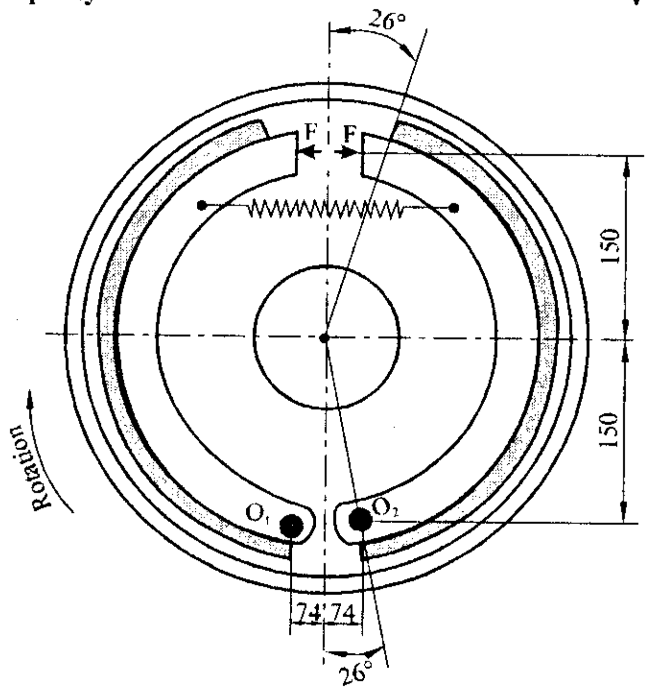


Fig. 6.45

Data :

$$r = \frac{400}{2} = 200 \text{ mm}; \; b = 40 \text{ mm}; \; \mu = 0.32; \; p_a = 1 \text{ MPa}$$

Solution :

i) **Actuating force**

$\theta_1 = 0°$ and $\theta_2 = 180 - 2 \times 26 = 128°$ [refer figure 4.40]

Distance between pivot and point of application of force $c = 150 + 150 = 300$ mm

Distance between pivot and drum centre $a = \sqrt{150^2 + 74^2} = 167.26$ mm

Maximum pressure occurs at an angle $\theta_a = 90°$ ($\because$ $\theta_2 > 90$)

Moment due to frictional force $M_\mu = \dfrac{\mu p_a br}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta(r - a\cos\theta)d\theta$ ---- 19.174

$$= \frac{\mu p_a br}{\sin\theta_a}\left[\int_{\theta_1}^{\theta_2} r\sin\theta d\theta - \int_{\theta_1}^{\theta_2} a\cos\theta\sin\theta d\theta\right]$$

$$= \frac{\mu p_a br}{\sin\theta_a}\left[r(-\cos\theta)_{\theta_1}^{\theta_2} - \int_{\theta_1}^{\theta_2} a\frac{\sin 2\theta}{2}d\theta\right]$$

$$= \frac{\mu p_a br}{\sin\theta_a}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta)_{\theta_1}^{\theta_2}\right]$$

$$= \frac{\mu p_a br}{\sin\theta_a}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

$$= \frac{0.32\times1\times40\times200}{\sin 90}\left\{200(\cos o - \cos 128) + \frac{167.26}{4}[\cos(2\times128) - \cos o]\right\}$$

$$\therefore\ M_{t\mu} = 694275.4\,\text{Nmm} = 694.2754\,\text{Nm}$$

Moment of normal force $M_{tn} = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta d\theta$ ---- 19.175

$$= \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \frac{p_a bra}{\sin\theta_a}\left[\int_{\theta_1}^{\theta_2}\frac{1}{2}d\theta - \int_{\theta_1}^{\theta_2}\frac{\cos 2\theta}{2}d\theta\right]$$

$$= \frac{p_a bra}{\sin\theta_a}\left[\frac{1}{2}(\theta)_{\theta_1}^{\theta_2} - \frac{1}{4}(\sin 2\theta)_{\theta_1}^{\theta_2}\right]$$

$$= \frac{p_a bra}{\sin\theta_a}\left[\frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right]$$

$$= \frac{1\times40\times200\times167.260}{\sin 90}\left\{\frac{1}{2}\left(128\times\frac{\pi}{180} - 0\right) - \frac{1}{4}[\sin(2\times128) - \sin 0]\right\}$$

i.e., $M_{tn} = 1819233\,\text{Nmm} = 1819.233\,\text{Nm}$

$\therefore$ Actuating force on the leading wheel $F = \frac{M_{tn} - M_{t\mu}}{c}$ --- 19.176

$$= \frac{1819233 - 694275.4}{300} = 3749.86\,\text{N}$$

Since same force on each shoe, actuating force on the trailing wheel F' = 3749.86 N

ii) Braking capacity

Torque applied to the drum by the leading brake shoe (Right shoe)

$$M_{tl} = \frac{\mu p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} \quad \text{---- 19.177}$$

$$= \frac{0.32 \times 1 \times 40 \times 200^2 [\cos o - \cos 128]}{\sin 90} = 827218.675 \text{ Nmm}$$

Normal pressure on the trailing shoe $p'_a = \frac{F.c.p_a}{M_{tn} + M_{t\mu}} = \frac{3749.86 \times 300 \times 1}{1819233 + 694275.4} = 0.4476 \text{ N/mm}^2$

∴ Torque applied to the drum by trailing brake shoe (left shoe)

$$M_{t_t} = \frac{\mu p'_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{M_{t_l}}{p_a} \cdot p'_a$$

$$= \frac{827218.675}{1} \times 0.4476 = 370234 \text{ Nmm}$$

∴ Total braking torque $M_t = M_{t_l} + M_{t_t}$

$$= 827218.675 + 370234 = 1197452.675 \text{ Nmm}$$

Example 6.41

Calculate the braking torque and power absorbed by an internal expanding shoe brake shown in Fig. 6.46 for the following data.

Speed of drum = 1000 rpm (ccw)

Angle between the pivot and the start (heel) of brake lining θ_1 = 30°

Angle between the pivot and the end (toe) of brake lining θ_2 = 135°

Force F on each shoe = 90 N

Coefficient of friction μ = 0.3

Face width of shoe b = 40 mm

Radius of brake drum r = 150 mm

Distance from the pivot to centre of drum a = 120 mm

Distance from the pivot to the point of application of force c = 200 mm

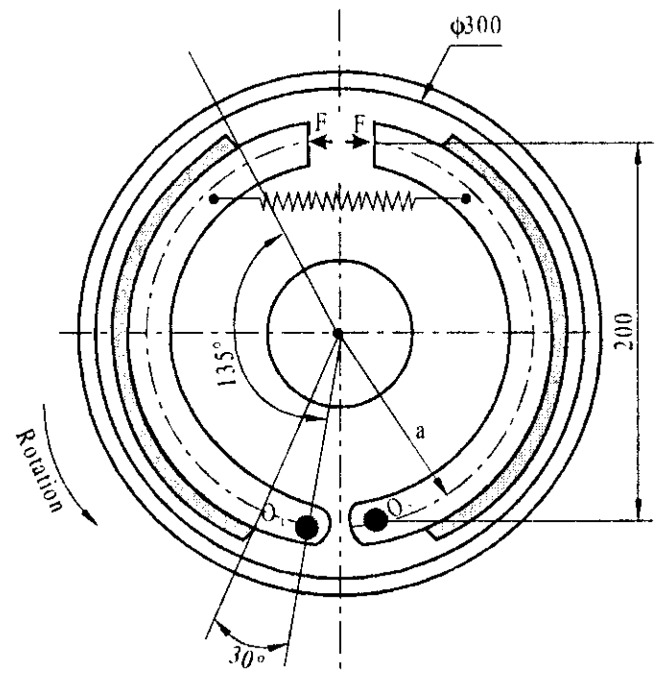


Fig. 6.46

Solution :

Maximum pressure occurs at an angle $\theta_a = 90°$ [$\because$ $\theta_2 > 90°$]

Moment due to frictional force $M_{t_\mu} = \dfrac{\mu p_a br}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2} \sin\theta\,(r - a\cos\theta)\,d\theta$ ---- 19.174

$$= \frac{\mu p_a br}{\sin\theta_a}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

$$= \frac{0.3\times p_a\times 40\times 150}{\sin 90}\left[150(\cos 30 - \cos 135) + \frac{120}{4}(\cos 270 - \cos 60)\right]$$

$$= 397745.7\,p_a$$

Moment due to normal force $M_{tn} = \dfrac{p_a bra}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2} \sin^2\theta d\theta$ ---- 19.175

$$= \frac{p_a bra}{\sin\theta_a}\left[\frac{1}{2}(\theta_2 - \theta_1) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right]$$

$$= \frac{p_a\times 40\times 150\times 120}{\sin 90}\left[\frac{1}{2}\left(135\times\frac{\pi}{180} - 30\times\frac{\pi}{180}\right) - \frac{1}{4}(\sin 270 - \sin 60)\right]$$

$$= 995619.03\,p_a$$

$\therefore$ Actual force on the leading wheel $F = \dfrac{M_{tn} - M_{t\mu}}{c}$

$$\text{i.e. } 90 = \frac{995619.03p_a - 397745.7p_a}{200}$$

$$\therefore\ p_a = 0.03\ \text{N/mm}^2$$

$\therefore$ Torque applied to the drum by the leading brake shoe (left shoe)

$$M_{t_l} = \frac{\mu p_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} \quad \text{—19.177}$$

$$= \frac{0.3 \times 0.03 \times 40 \times 150^2[\cos 30 - \cos 135]}{\sin 90}$$

$$= 12742.4\ \text{Nmm}$$

Normal pressure on the trailing shoe $p'_a = \dfrac{F.c.P_a}{M_{tn} + M_{t\mu}}$

$$= \frac{90 \times 200 \times 0.03}{(995619.03 + 397745.7)0.03} = 0.01292\ \text{N/mm}^2$$

$\therefore$ Torque applied to the drum by the trailing brake shoe (right shoe)

$$M_{t_t} = \frac{\mu p'_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{M_{tl}}{p_a} \times p'_a = \frac{12742.4}{0.03} \times 0.01292 = 5487\ \text{Nmm}$$

$\therefore$ Total braking torque $M_t = M_{t_l} + M_{t_t} = 12742.4 + 5487 = 18229.4$ Nmm

$$\text{Also } M_t = 9550 \times 1000 \times \frac{N}{n} \text{ where } M_t \text{ in Nmm}$$

$$\text{i.e., } 18229.4 = 9550 \times 1000 \times \frac{N}{1000}$$

$\therefore$ Power absorbed $N = 1.909$ kW

REVIEW QUESTIONS

1. Explain briefly the uniform pressure theory and uniform wear theory as applicable to friction clutches and brakes. **VTU, February 2002**
2. Name the different type of clutches. Describe with the help of a neat sketch the working principles of any one friction clutch. **VTU, February 2002 (IM/IP)**
3. Classify the brakes and name different types of mechanical brakes. **BU, February/March 2001**
4. Derive a relation to compute the torque developed on block brake. **BU, February/March 2001**

EXERCISES

1. Determine the torque that may be resisted by the single block brake shown in Fig. 6.47.

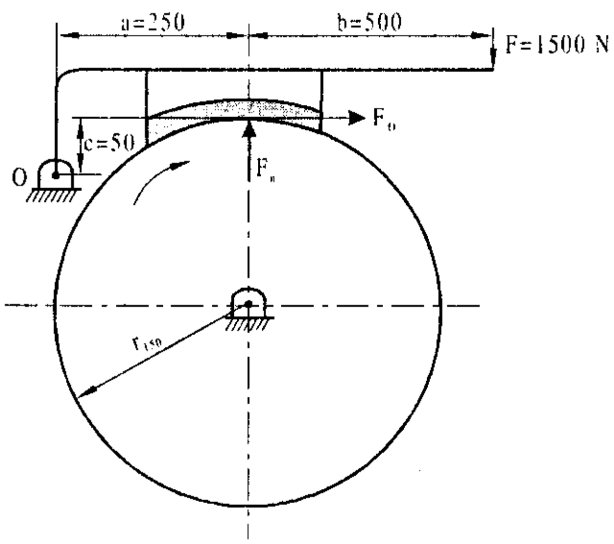


Fig. 6.47

2. An automotive single plate clutch consists of two pairs of contacting surfaces. The outer diameter of the friction disc is 270 mm. The coefficient of friction is 0.3 and the maximum intensity of pressure is 0.3 MPa. The clutch is transmitting a torque of 530 Nm. Assuming uniform wear calculate (i) The inner diameter of the friction disc and (ii) Spring force required to keep the clutch engaged.
3. An internal expanding brake has a inner surface of rim of diameter 500 mm. The distance between the fulcrums is 100 mm. The distance between the fulcrum and the point of application of efforts is 400 mm. The brake linings sustain an angle of 120° at the centre. The material

of the lining has the coefficient of friction of 0.3 and an allowable bearing pressure of 0.5 MPa. Determine

i) Effort required to stop the rotation of the brake drum.

ii) Width of the brake lining.

The brake transmits a power of 30 kW at a rated speed of 1500 rpm.

4. Design a cone clutch to transmit 12 kW at 1000 rpm, selecting suitable material. Also determine the axial force necessary to engage the cone clutch.

5. A multi disc clutch has three discs on the driving shaft and two on the driven shaft. The inside diameter of the contact surface is 120 mm. The maximum pressure between the surfaces is limited to 0.1 MPa. Design the clutch for transmitting 25 kW at 1515 rpm. Assume uniform wear condition and $\mu = 0.3$.

6. Design a centrifugal clutch with four shoes for transmitting 20 kW at 1200 rpm. The speed at which engagement begins is 80% of the running speed. The inside radius of the pulley rim is 150 mm. The shoes are lined with Ferodo lining for which $\mu = 0.25$.

7. Design a single plate clutch to transmit a power of 30 kW at a rated speed of 1500 rpm. Space limits the outer diameter of the clutch lining to 180 mm. Select suitable material for the friction lining. Also determine the axial force necessary to engage the clutch. Use uniform wear theory for design.

8. a) Design a cone clutch to transmit a power of 40 kW at a rated speed of 750 rpm. Also determine the

   i) The axial force capacity

   ii) The axial force necessary to transmit the torque

   iii) The axial force necessary to engage the cone clutch.

   b) A simple band brake shown in Fig.6.48 is to be designed to absorb a power of 30 kW at a rated speed of 750 rpm. Determine :

   i) The effort required to stop clockwise rotation of the brake drum.

   ii) The effort required to stop counter clockwise rotation of the brake drum.

   iii) The dimensions of the rectangular cross-section of the brake lever assuming its depth to be twice the width.

   iv) The dimension of the cross-section of the band assuming its width to be ten times the thickness.

VTU, Jan/Feb 2005

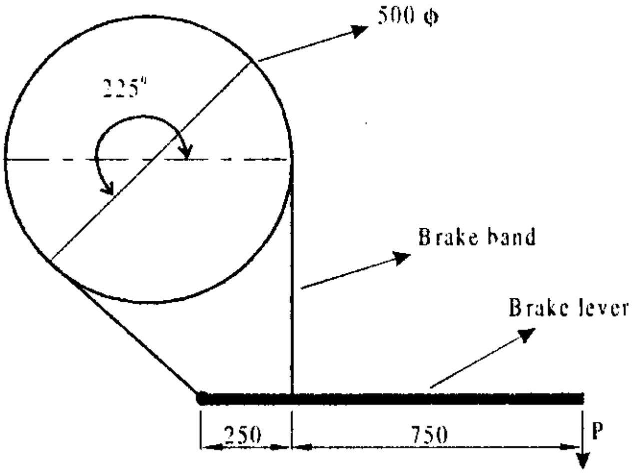


Fig. **6.48**

9. a) Design a cone clutch to transmit a power of 40 kW at a rated speed of 750 rpm. Also determine
   i) Axial force necessary to transmit torque
   ii) Axial force necessary to engage the cone clutch.

   b) A simple band brake shown in Fig.6.49 is to be designed to stop the rotation of a shaft transmitting a power of 45 kW at rated speed of 500 rpm. Selecting suitable materials determine
   i) Dimensions of the rectangular cross section of the band
   ii) Dimensions of the rectangular cross section of the brake lever.
   iii) Diameter of the fulcrum pin. **VTU, Jan/Feb 2006**

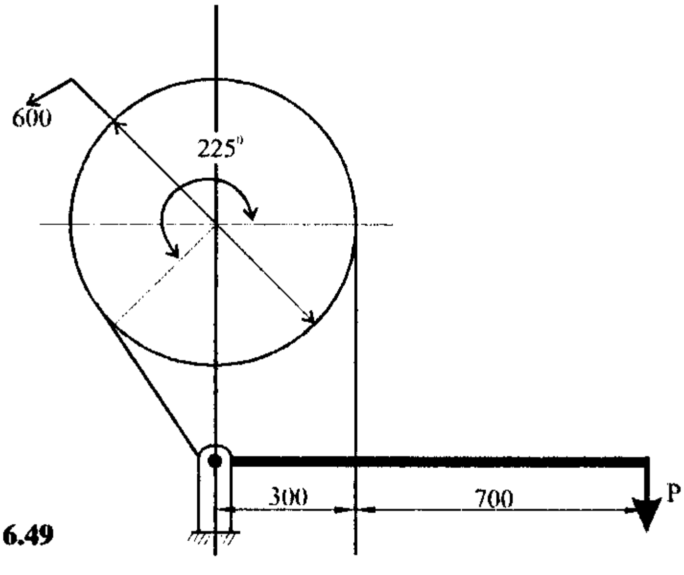


Fig. **6.49**